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Strong Accumulators from Collision-Resistant Hashing

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Outline

- Basic Cryptographic Concepts
- Notion of Cryptographic Accumulator
- Our construction [CHKO08]
- Conclusion

Basic Cryptographic Concepts

- How to define security?
  - This is one of the cryptographer's hardest task.
  - A good definition should capture intuition… … and more!
  - Community had to wait until 1984 with [GM84] for a satisfactory definition of (computational) *"secure encryption"*.

Basic Cryptographic Concepts

- Adversary
  - With unlimited computational power
    - One Time Pad, Secret Sharing
  - Computationally Bounded
    (Probabilistic Polynomial Time = PPT)
    - Key Agreement, Public Key Encryption, Digital Signatures, Hash Functions, Commitments,…

Basic Cryptographic Concepts

- Cryptographic Assumptions
  - Most of cryptographic constructions rely on **complexity assumptions**.
    - Factoring is hard.
    - Computing Discrete Logarithm is hard.
    - Existence of functions with “good” properties
      - One-way functions
      - Collision-Resistant Hash functions
    - …
  - All these assumptions require that $P \neq NP$.
  - Some assumptions are implied by others.

Basic Cryptographic Concepts

- How to prove security?
  - What we want:
    - Assumption X holds (for any adversary) => protocol P is secure.
    - No *adversary* can break X => No *adversary* can break P.
  - What we do:
    - Suppose protocol P is insecure => X does not hold.
    - Let *A* the *adversary* that breaks P => We can build an *adversary* *B* that breaks X.
  - This method is sometimes called *"Provable Security"* or *"Reductionist Security"*.

Basic Cryptographic Concepts

- **Let's get into the details…**
  - We need to quantify the probability that an *adversary* can compute some values.
  - **Asymptotic notion**
    - The running time of the adversary depends on the ***security parameter***.
    - **E.g:** size of the secret key in the case of encryption, size of the primes for the factoring assumption.
  - **Definition:** (negligible function)
    A function $\varepsilon : \mathbf{N} \rightarrow [0,1]$ is negligible if for
    <u>*every*</u> *polynomial* $q: \mathbf{N} \rightarrow \mathbf{N}$, for k sufficiently large:
    $$\varepsilon(k) < |1/q(k)|$$

Basic Cryptographic Concepts

- RSA
  - Initialization
    - $n=pq$, p,q safe primes , $\Phi(n) = (p-1)(q-1) = |Z_n^*|$
    - $e \in Z_{\Phi(n)}^*$ (encryption)
    - $d \in Z_{\Phi(n)}^*$ (decryption)
    - $ed = 1 \bmod \Phi(n)$ (*Euclidian Algorithm*)
  - Encryption / Decryption
    - $x \in Z_n^*$ plaintext
    - Encrypt: $c = x^e \bmod n$
    - Decrypt: $y = c^d \bmod n = x^{ed} \bmod n = x \bmod n$

Basic Cryptographic Concepts

- Assumptions
  - *RSA Instance generator*

    $(n,p,q,e,d) \leftarrow I(k)$

  - *Factoring Assumption*

    $Pr\,[(p,q) \leftarrow A(n) : n=pq] < \varepsilon(k)$

  - *RSA Assumption*

    $Pr\,[y \in_R Z_n^* \,;\, x \leftarrow A(n,y,e) : y=x^e \bmod n] < \varepsilon(k)$

  - *Strong RSA Assumption [BarPfi97]*

    $Pr\,[u \in_R Z_n^* \,;\, (x,e) \leftarrow A(n,u) : u=x^e \bmod n,\ e \neq 1] < \varepsilon(k)$

  - *Strong RSA => RSA => Factoring*

    (note the direction <= is open)

Basic Cryptographic Concepts

- Assumptions and efficiency
  - We know how to build encryption schemes based on
    - RSA Assumption
    - Factoring Assumption
  - However encryption algorithms based on the **RSA Assumption** are much *faster* than those based only on the **Factoring Assumption**.

Basic Cryptographic Concepts

- Collision-Resistant Hash Functions
  - $H:\{0,1\}^* \rightarrow \{0,1\}^k$
    - Given x, it is *easy* to compute $H(x)$.
    - Given y, *hard* to compute x such that $H(x)=y$.
    - Given x, *hard* to compute $x' \neq x$ such that $H(x)=H(x')$.
    - *Hard* to compute $x \neq x'$ such that $H(x)=H(x')$.

This definition is not formal. Just an intuition.

Basic Cryptographic Concepts

- Formal definition for
  Collision-Resistant Hash Functions
  - **Definition:** (1st attempt)
    A function H is collision-resistant iff:
    For all A: $\Pr[x,x' \leftarrow A():x \neq x' \text{ and } H(x)=H(x')] < \varepsilon(k)$
  - Why does the previous definition not work?
    - A():
      return (x,x') // Where (x,x') is a collision-pair

Basic Cryptographic Concepts

- **Definition:**
  (family of collision-resistant hash functions)
  - $\{F_k\}_{k \in N}$ where $F_k=\{Hj, j \in J_k\}$ is a family of collision resistant hash functions iff:
    - For all j, H_j can be selected efficiently,
    - $Pr_{j \in J_k}[x,x' \leftarrow A(j,k): x \neq x' , H_j(x)=H_j(x')] < \varepsilon(k)$

Basic Cryptographic Concepts

- **Assumption:**
  Collision-Resistant Hash Functions (CRHF) exist.

Outline

- Basic Cryptographic Concepts
- Notion of Cryptographic Accumulator
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Notion of Cryptographic Accumulator

- Problem
  - A set X.
  - Given an element x we wish to prove that this element belongs or not to X.
- Let $X=\{x_1,x_2,\ldots,x_n\}$:
  - X will be represented by a short value Acc.
    - Acc is the *Accumulated Value*
  - Given x and w (*witness*) we want to check whether x belongs to X.

Notion of Cryptographic Accumulator

- Participants
  - Manager
    - Computes the accumulated value …
    - … and the witnesses.
  - User
    - Tests for (non)membership of a given element using the accumulated value and a witness provided by the manager.

Properties

- Dynamic
  - Allows insertion/deletion of elements.
- Universal
  - Allows proofs of membership and nonmembership.
- Strong
  - No need to trust in the Accumulator Manager.

Prior work

	Dynamic	Strong	Universal	Security	Efficiency (witness size)	Note
[BeMa94]	✗	✓	✗	RSA + RO	O(1)	First definition
[BarPfi97]	✗	✓	✗	Strong RSA	O(1)	-
[CamLys02]	✓	✗	✗	Strong RSA	O(1)	First dynamic accumulator
[LLX07]	✓	✗	✓	Strong RSA	O(1)	First universal accumultor
[AWSM07]	✓	✗	✗	Pairings	O(1)	E-cash

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[AWSM07]	✓	✗	✗	Pairings	O(1)	E-cash
[CHKO08]	✓	✓	✓	Collision-Resistant Hashing	O(ln(n))	Our work

Some Applications

- Time-Stamping [BeMa94]
- Anonymous Credentials [CamLys02]
- Broadcast Encryption [GeRa04]
- Certificate Revocation List [LLX07]
- E-Cash [AWSM07]
- Electronic Invoice Factoring [CHKO08]

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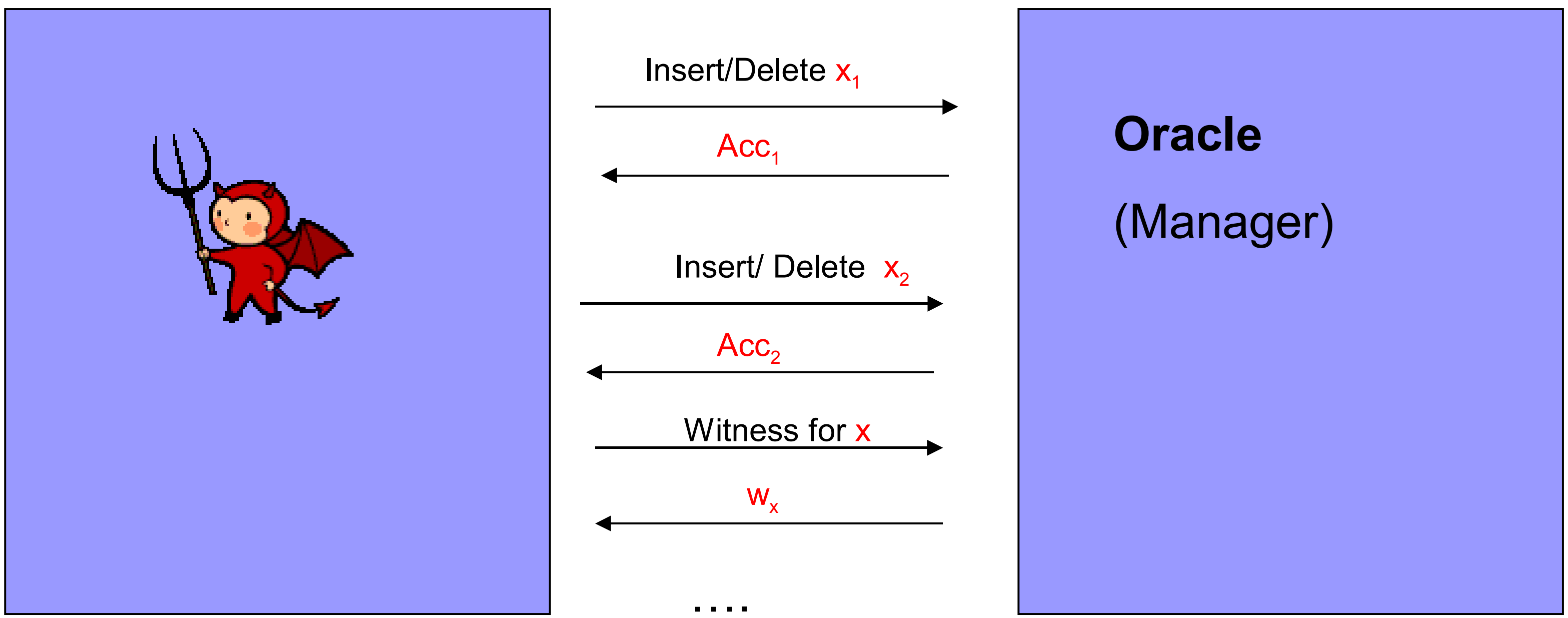
Dynamic Accumulators [CamLys02]

- Security Model



Scheme secure iff:

$\Pr[(w,x,X) \leftarrow A^{O}(): \mathrm{Belongs}(w,x,Acc)=1 \text{ and } x \notin X] < \varepsilon(k)$

Dynamic Accumulators [CamLys02]

- Initialization
  - $n = pq$, $u \in Z_n^*$
- Set
  - $X=\{x_1,x_2,\ldots,x_l\}$ (primes)
- Accumulated value
  - $Acc = u^{x_1 \cdot x_2 \cdots x_l} \bmod n$
- Witness for x_i
  - $w = u^{x_1 \cdots x_{i-1} \cdot x_{i+1} \cdots x_l} \bmod n$
- Membership test
  - $w^{x_i} \bmod n = Acc$

Dynamic Accumulators [CamLys02]

- **To add elements**
  - $Acc' := Acc^x \bmod n$
  - $w' := w^x \bmod n$

- **To delete elements**
  - Recompute the accumulated value with all the elements of the new set.
  - Doing the same for the witnesses (without the element we want to test).
  - $O(|X|)$ => **NOT EFFICIENT**

- **To delete elements efficiently**
  - Manager knows $\Phi(n)$, x to be deleted
    - $Acc = u^{x_1 \cdot x_2 \cdot \ldots x \ldots x_l} \bmod n$
    - Compute $y = x^{-1} \bmod \Phi(n)$
    - $Acc_{new} = Acc^{1/x} \bmod n = Acc^y \bmod n$
  - The manager **must be trusted** because he can compute fake witnesses for any x
    - $w = Acc^{1/x} \bmod n$

Dynamic Accumulators [CamLys02]

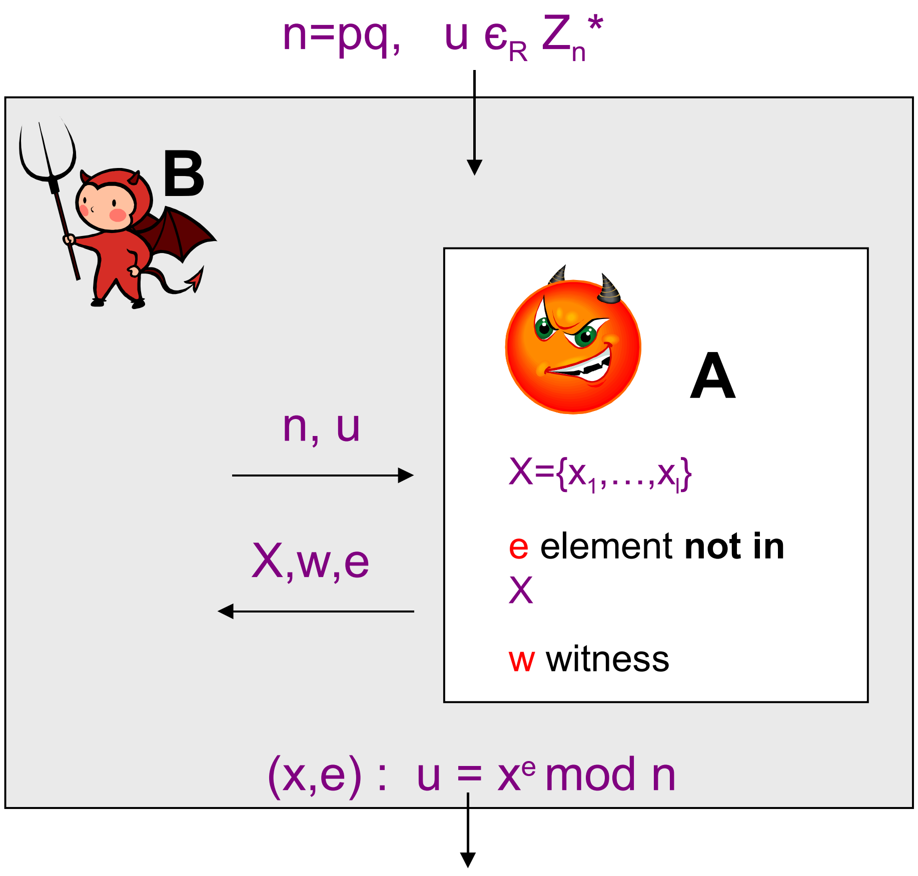
- **Theorem:** if the Strong RSA Assumption holds, the dynamic accumulator is secure.

Dynamic Accumulators [CamLys02]

- **Lemma:** Let n be an integer, given $u,v \in Z_n^*$ and $a,b \in Z$ such that $u^a = v^b \bmod n$ and $\gcd(a,b) = 1$, we can compute efficiently $x \in Z_n^*$ such that $x^a = v \bmod n$.
- **Proof:**
  - $\gcd(a,b)=1 \Rightarrow bd = 1 + ac$
  - $x := u^d v^{-c} \Rightarrow x^a = u^{da} v^{-ca} = (u^a)^d v^{-ca} = v^{bd} v^{-ca} = v$

Dynamic Accumulators [CamLys02]

- Proof of the theorem:



If there exists an adversary **A** that can break our scheme

We can build an adversary **B** that can break the Strong RSA Assumption

Dynamic Accumulators [CamLys02]

- Proof of the theorem:
  - $X = \{x_1,\ldots,x_l\}$
  - $Acc = u^{x_1 \cdots x_l} \bmod n = u^v \bmod n$
  - e does not belong to X
  - $w^e \bmod n = Acc = u^v \bmod n$
  - $gcd(v,e) = 1$ and $w^e = u^v \bmod n$

    => by the lemma we can conclude

Outline

- Basic Cryptographic Concepts
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- Constructions
  - Dynamic Accumulators [CamLys02]
  - Our Construction [CHKO08]
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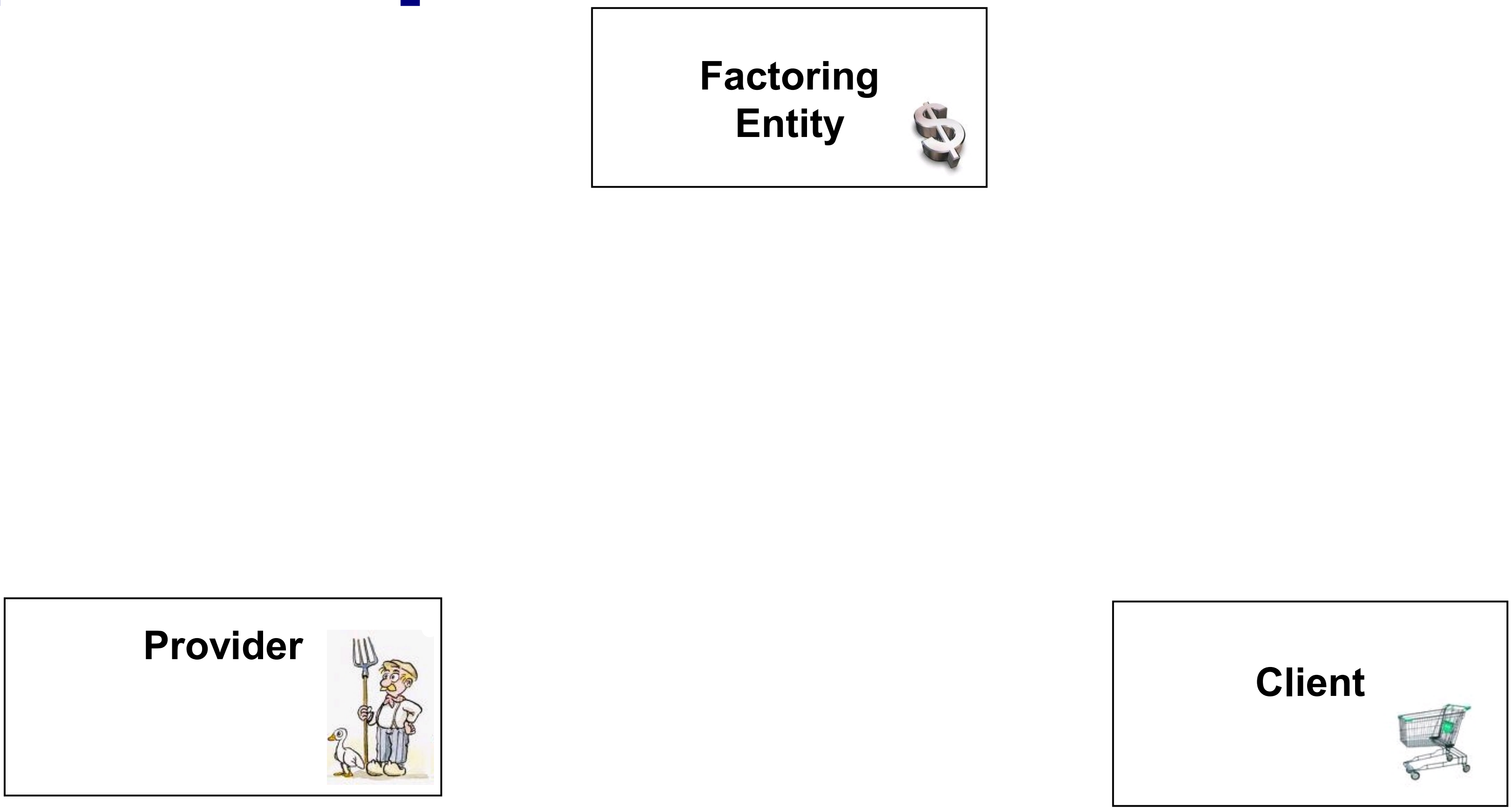
Not related to Number Theory!

Factoring Industry in Chile [CHKO08]



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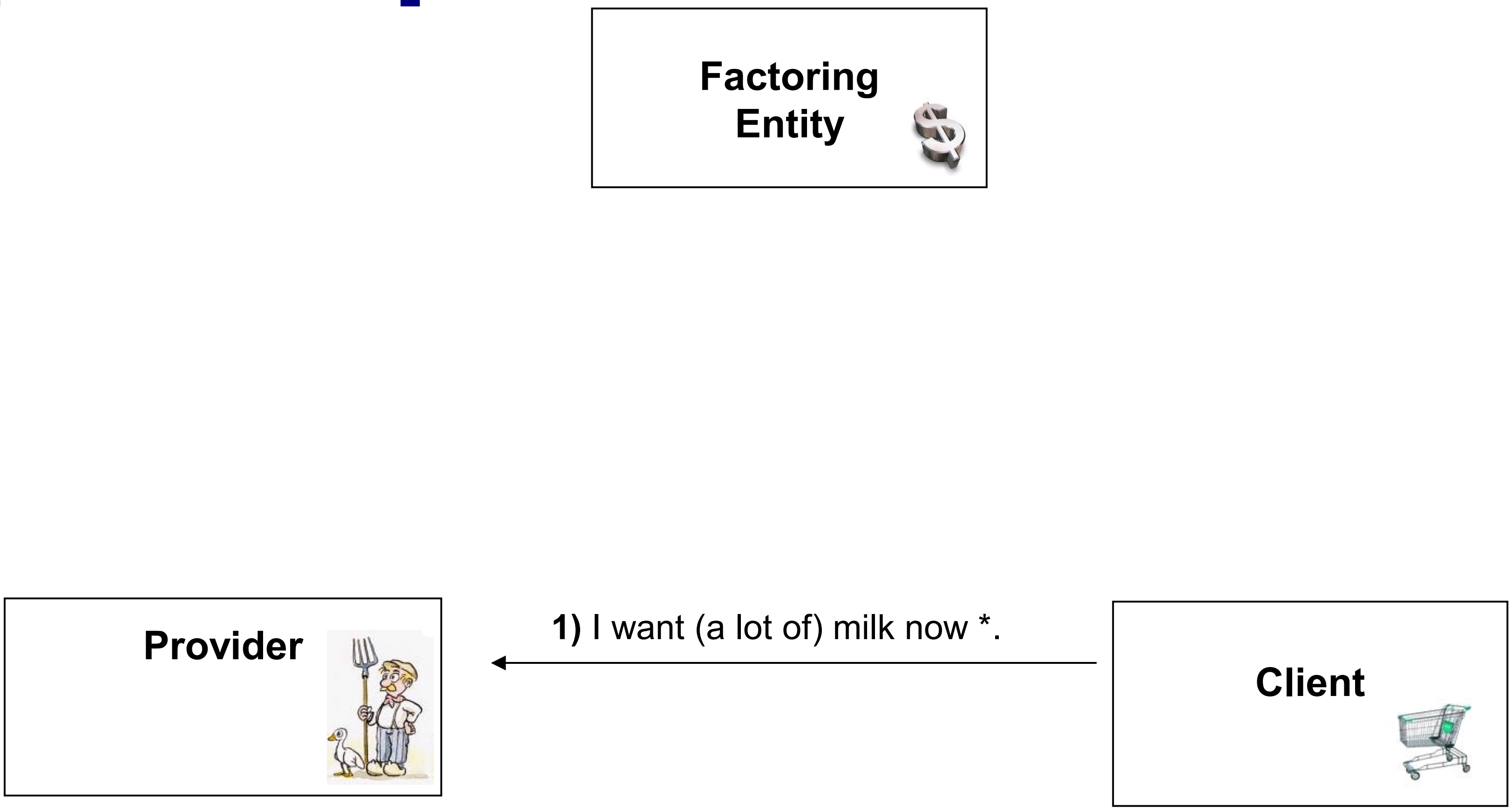

(*) but I do not want to pay yet.

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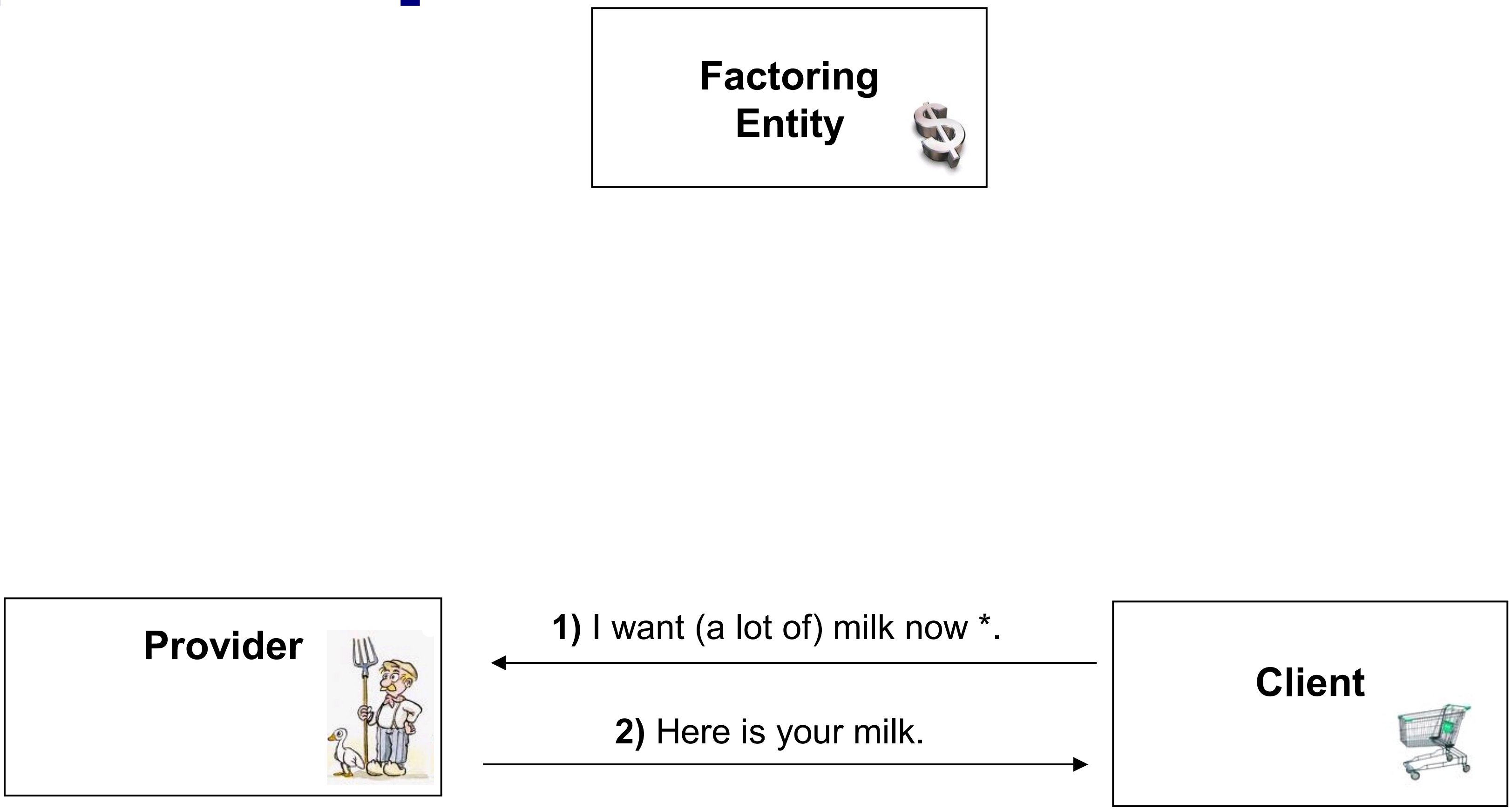

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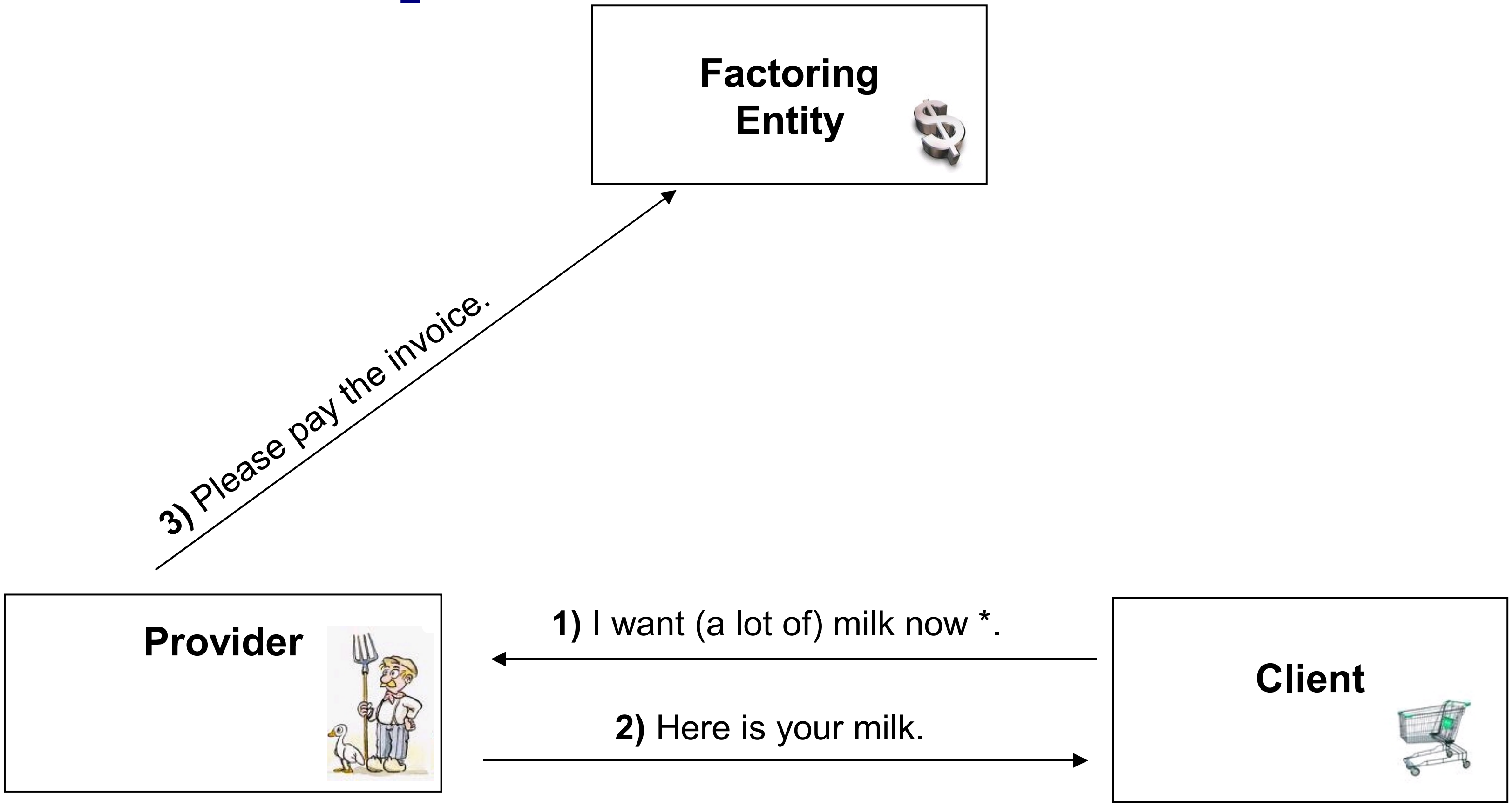

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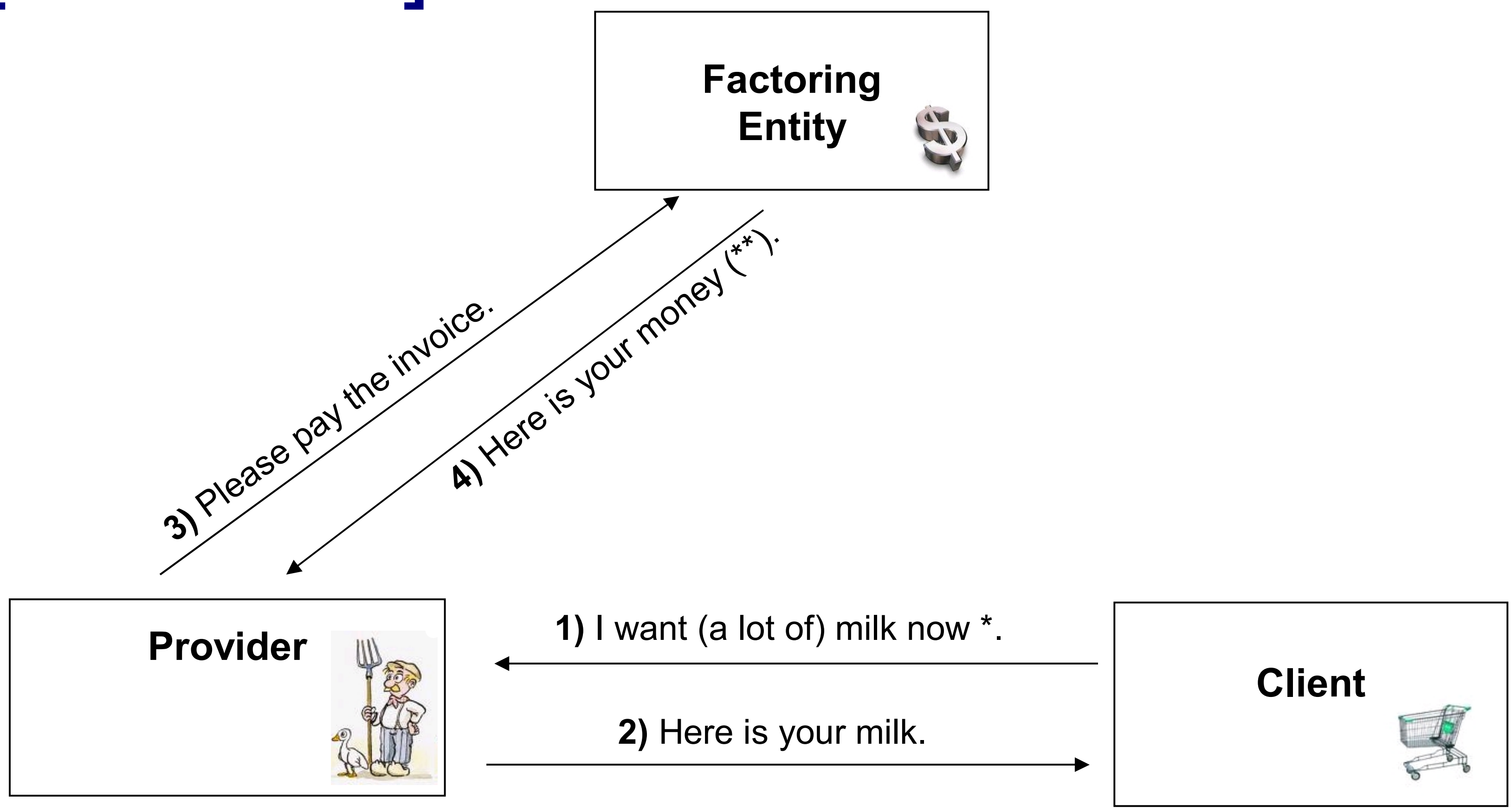

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(**) minus a fee.

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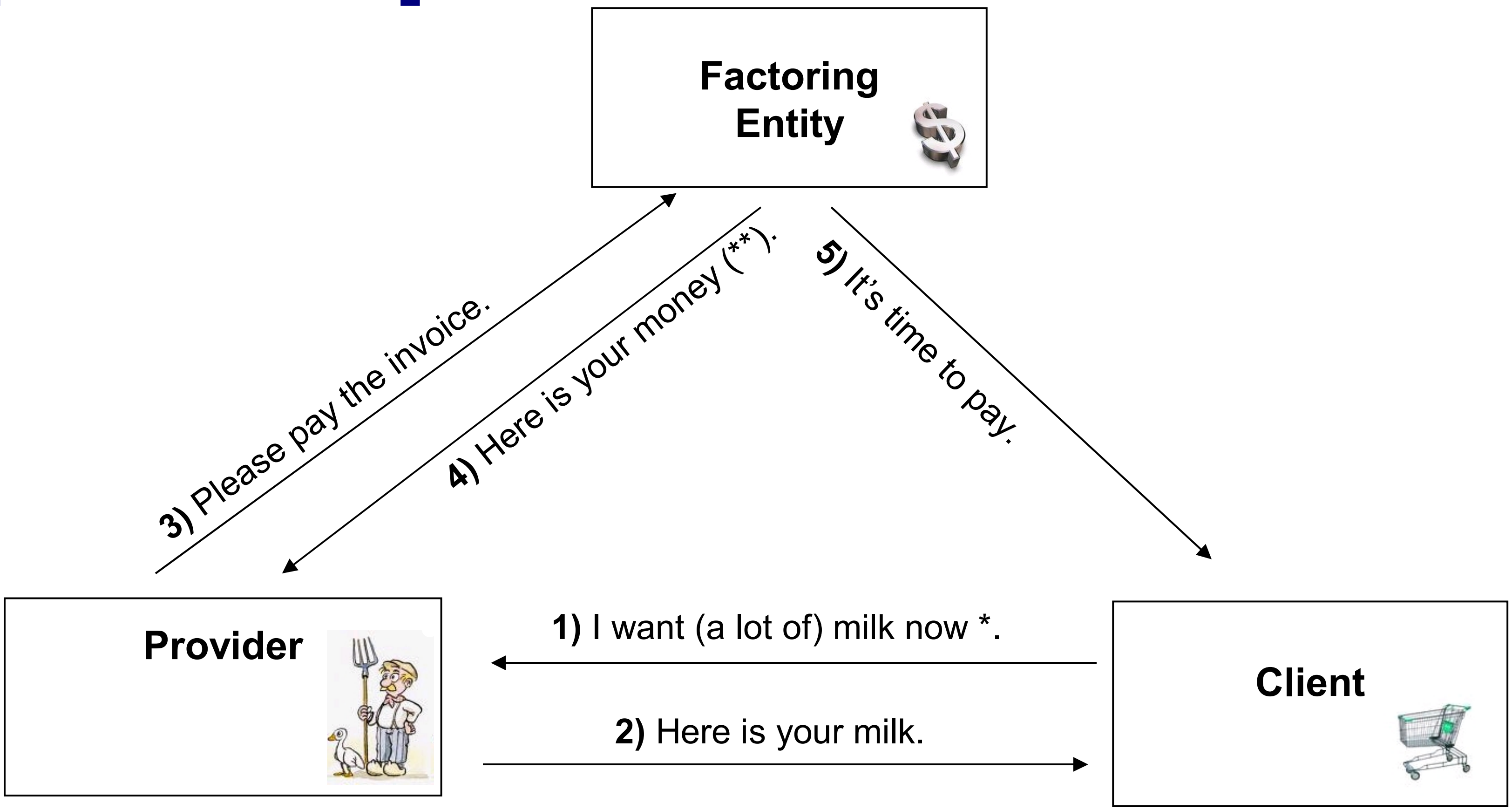

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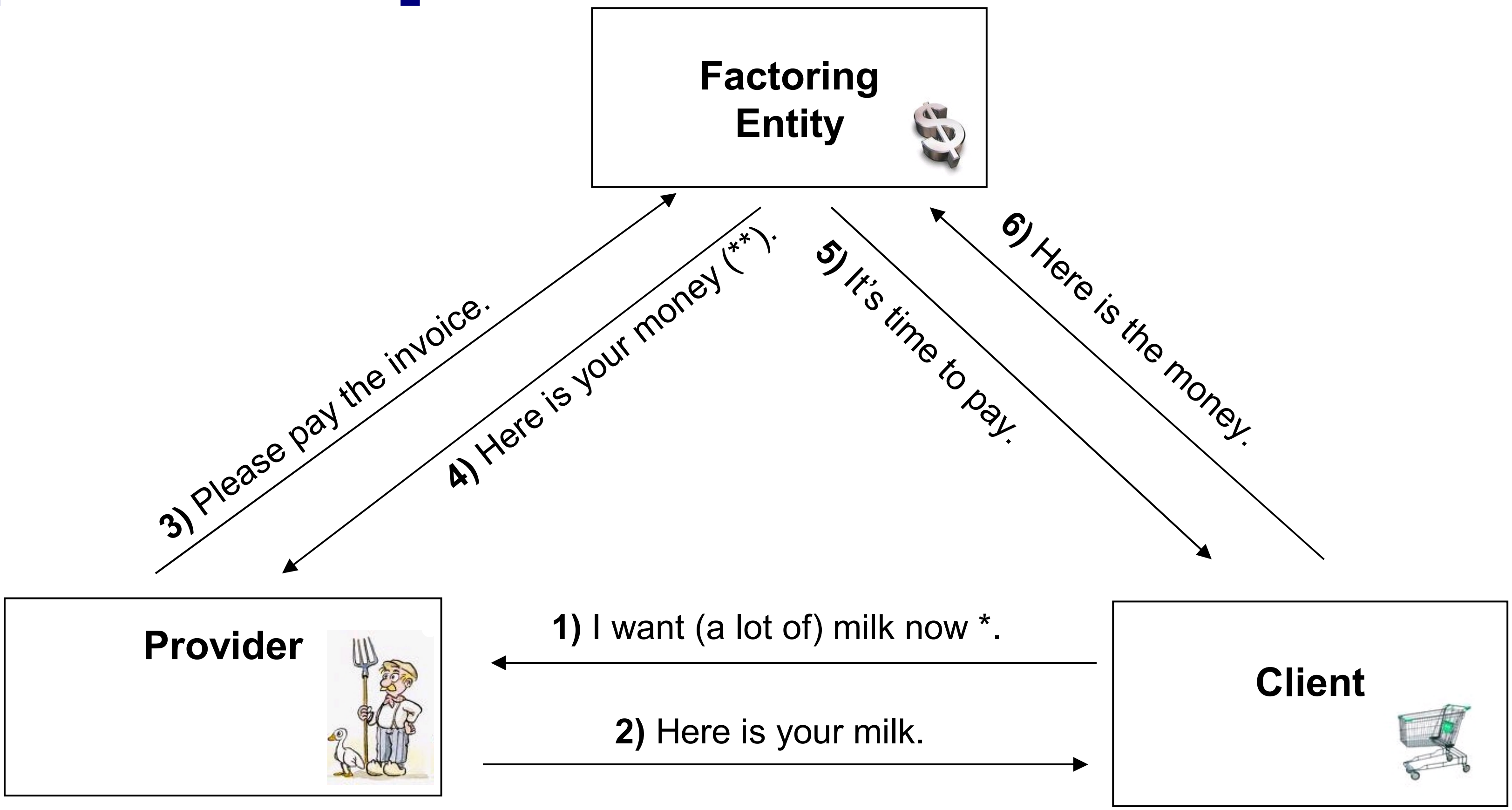
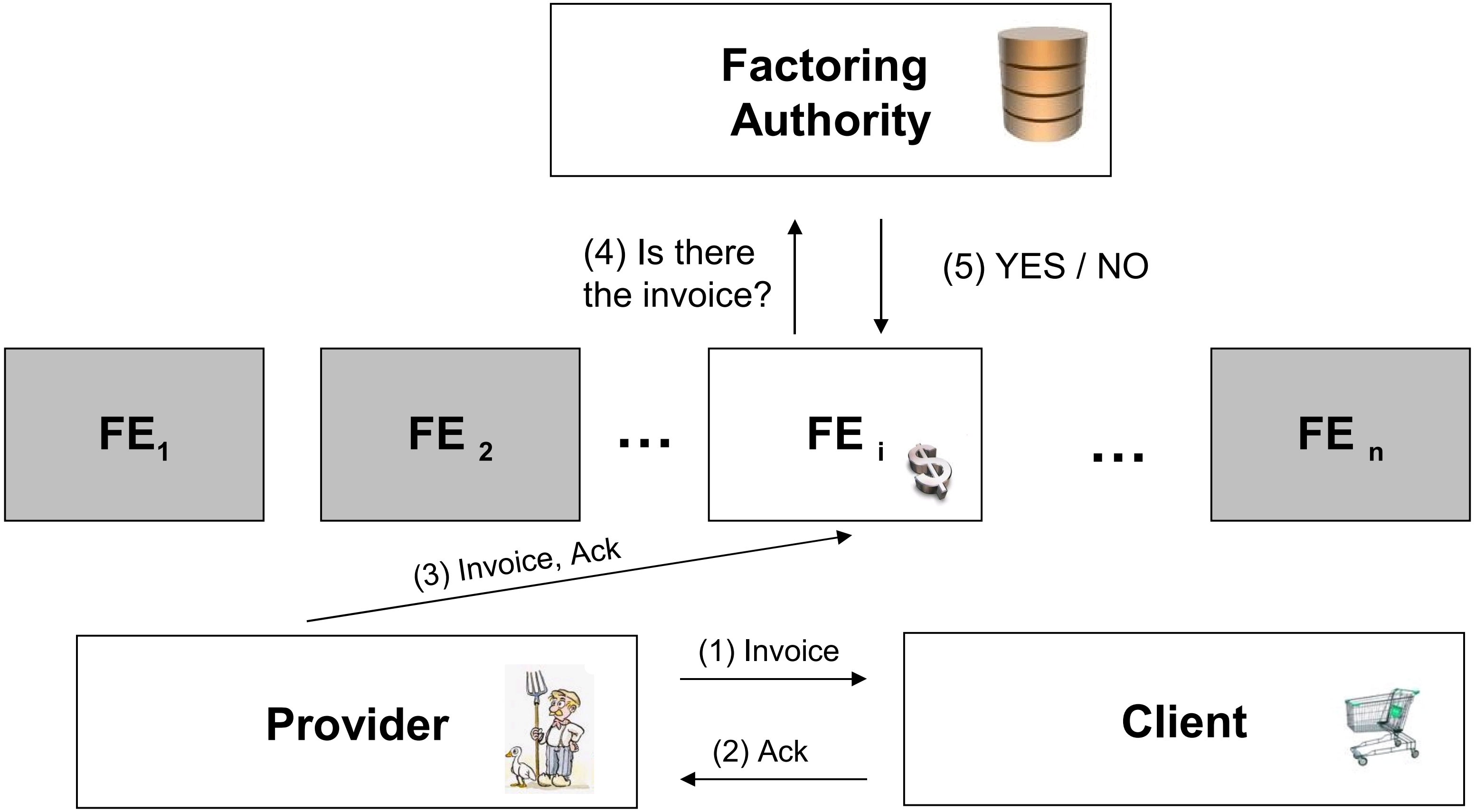


(*) but I do not want to pay yet.
(**) minus a fee.

The Problem

- A malicious provider could send the same invoice to various Factoring Entities.
- Then he leaves to a far away country with all the money.
- Later, several Factoring Entities will try to charge the invoice to the same client. Losts must be shared…

Solution with Factoring Authority



Caveat

- This solution is quite simple.
- **However**
  - Trusted Factoring Authority is needed.
- Can we remove this requirement?

Notation

- $H: \{0,1\}^* \rightarrow \{0,1\}^k$
  - Collision-resistant hash function
- $x_1, x_2, x_3, \ldots \in \{0,1\}^k$
  - $x_1 < x_2 < x_3 < \ldots$ where $<$ is the lexicographic order on binary strings.
- $-\infty, \infty$
  - Special values such that
    - For all $x \in \{0,1\}^k$: $-\infty < x < \infty$
- $||$ denotes the concatenation operator.

Public Data Structure

- Manager owns a public data structure called "Memory".
- Compute efficiently the accumulated value and the witnesses.
- In our construction the Memory M will be a binary tree.

Accumulator Operations

Operation	Who runs it?
$Acc_0, M_0 \leftarrow$ Setup(1^k)	Manager
$w \leftarrow$ Witness(M,x)	Manager
True,False,$\perp \leftarrow$ Belongs(x,w,Acc)	User
$Acc_{after}, M_{after}, w_{up} \leftarrow Update_{add/del}(M_{before}, x)$	Manager
OK, $\perp \leftarrow$ CheckUpdate($Acc_{before}, Acc_{after}, w_{up}$)	User

Checking for (non)membership

User

Does x belong to X? →

Accumulator Manager

w = Witness(M,x)

← w

Belongs(x,w,Acc) = True ⇔ x є X

Update of the accumulated value

User

$CheckUpdate(Acc_{before}, Acc_{after}, w_{up})$

Insert or Delete x →

← Acc_{after}, w_{up}

Accumulator Manager

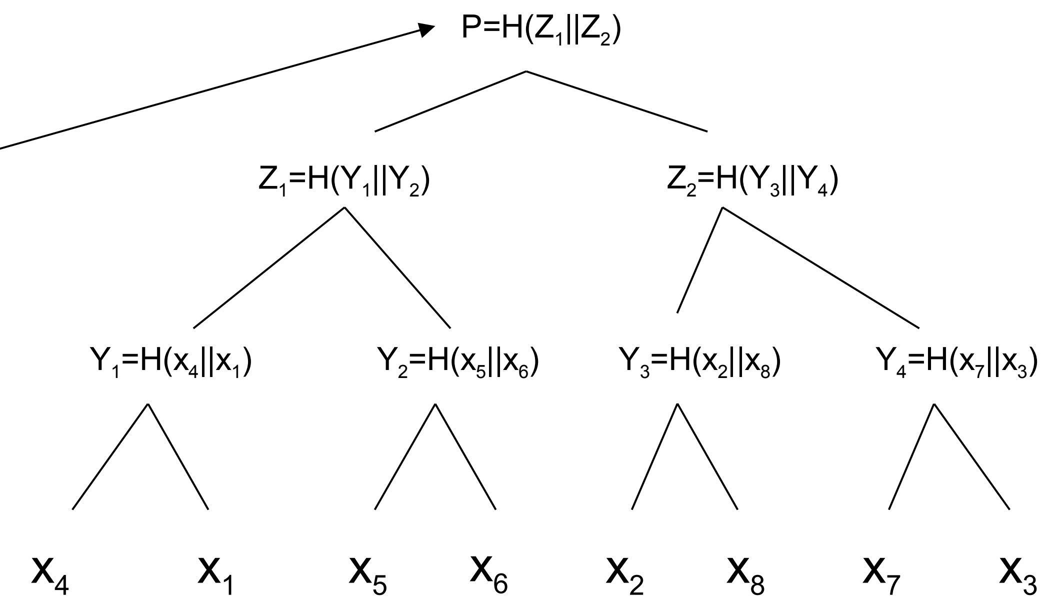
$Acc_{after}, M_{after}, w_{up} = Update_{Add/Del}(M_{before}, x)$

Ideas

- Merkle-trees

Root value:

Represents the set $\{x_1,\ldots,x_8\}$

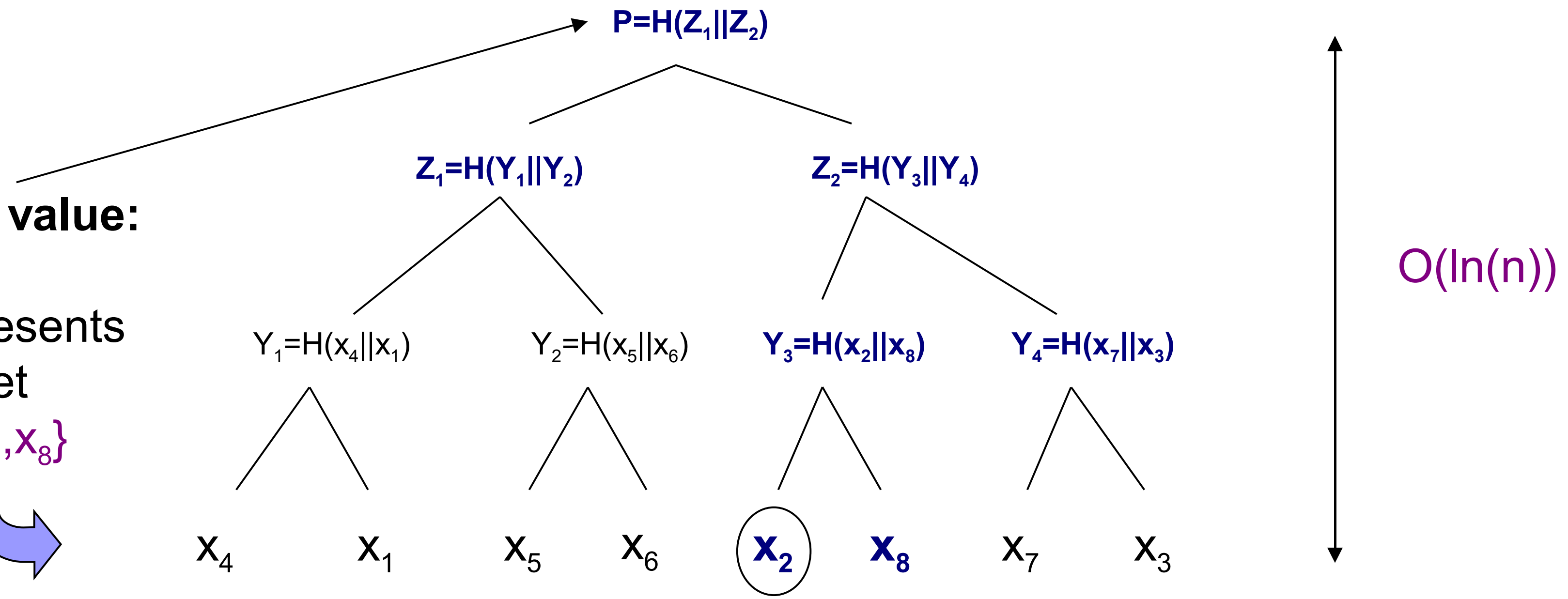


Ideas

- Merkle-trees

Root value:

Represents the set $\{x_1,...,x_8\}$



Ideas

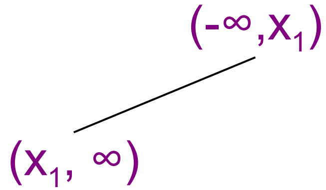
- How to prove nonmembership?
  - Kocher's trick [Koch98]: store pair of consecutive values
    - $X=\{1,3,5,6,11\}$
    - $X'=\{(-\infty,1),(1,3),(3,5),(5,6),(6,11),(11,\infty)\}$
    - $y=3$ belongs to X ⇔ $(1,3)$ or $(3,5)$ belongs to X'.
    - $y=2$ does not belong to X ⇔ $(1,3)$ belongs to X'.

How to insert elements?

(-∞,∞)

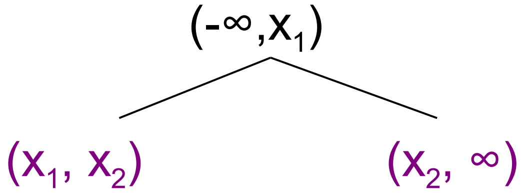
X=Ø, next: x_1

How to insert elements?



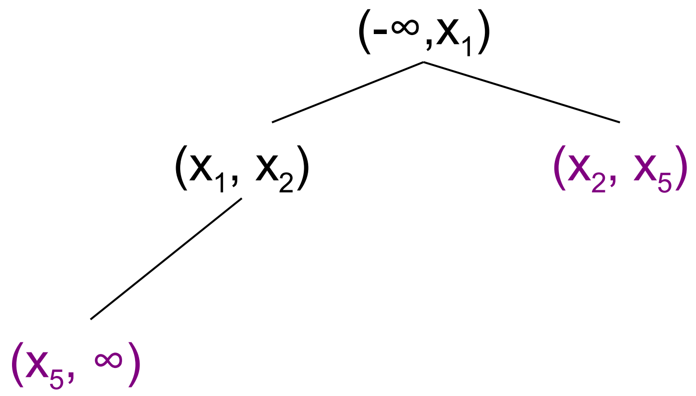
$X=\{x_1\}$, next: x_2

How to insert elements?



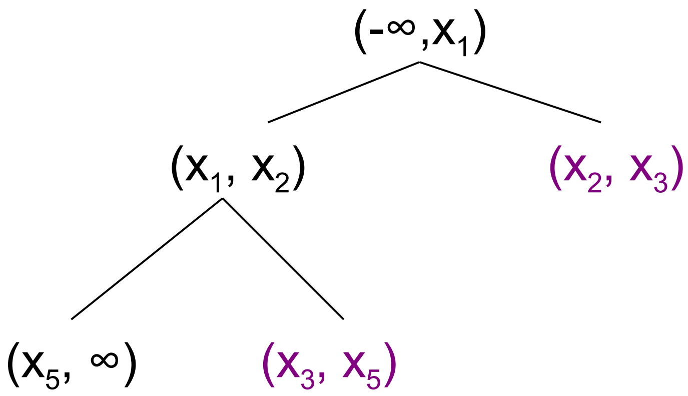
$X=\{x_1,x_2\}$, next: x_5

How to insert elements?



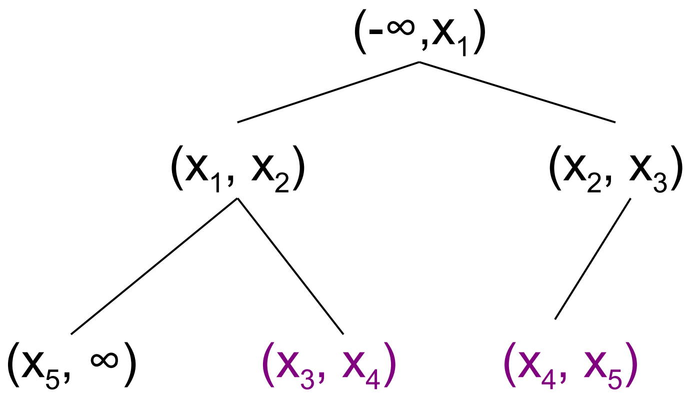
$X=\{x_1, x_2, x_5\}$, next: x_3

How to insert elements?



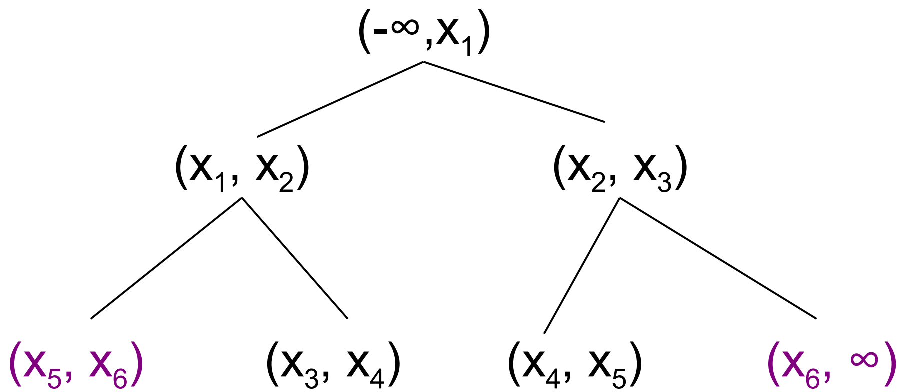
$X=\{x_1,x_2,x_3,x_5\}$, next: x_4

How to insert elements?



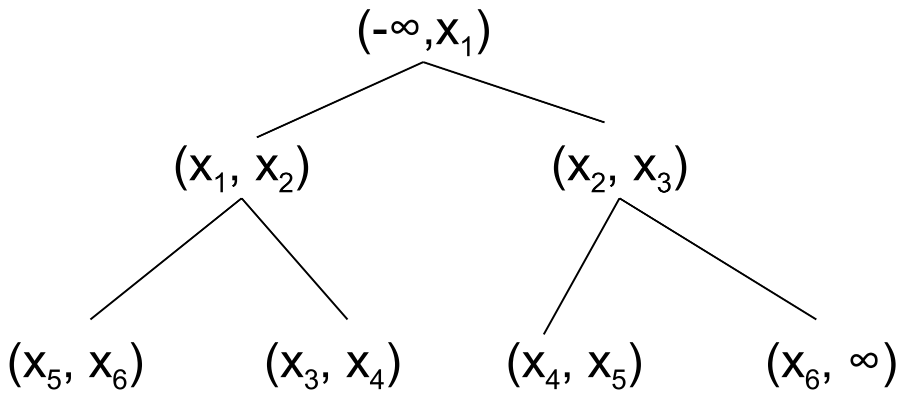
X={x_1,x_2,x_3,x_4,x_5}, next: x_6

How to insert elements?



$X=\{x_1,x_2,x_3,x_4,x_5,x_6\}$

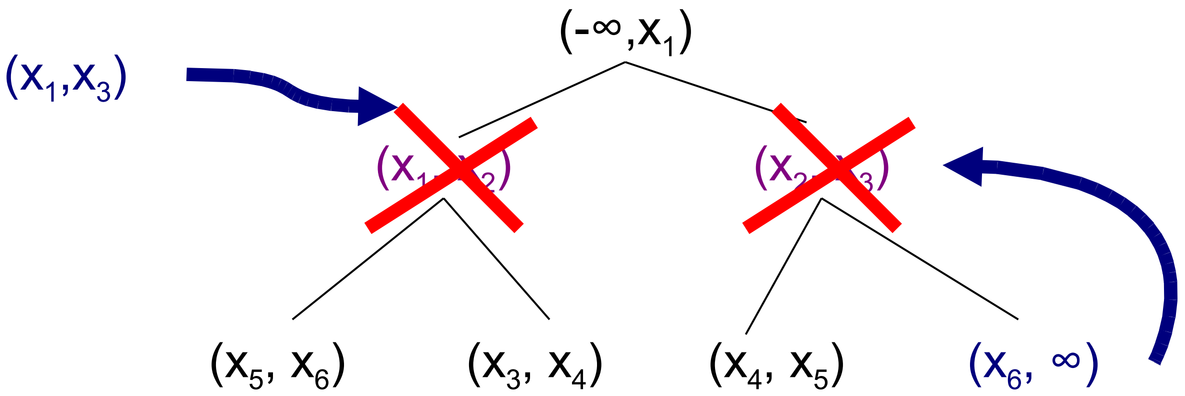
How to delete elements?



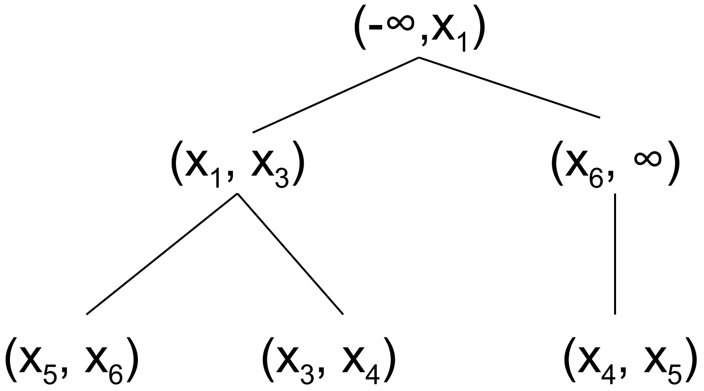
$X=\{x_1,x_2,x_3,x_4,x_5,x_6\}$

element to be deleted: x_2

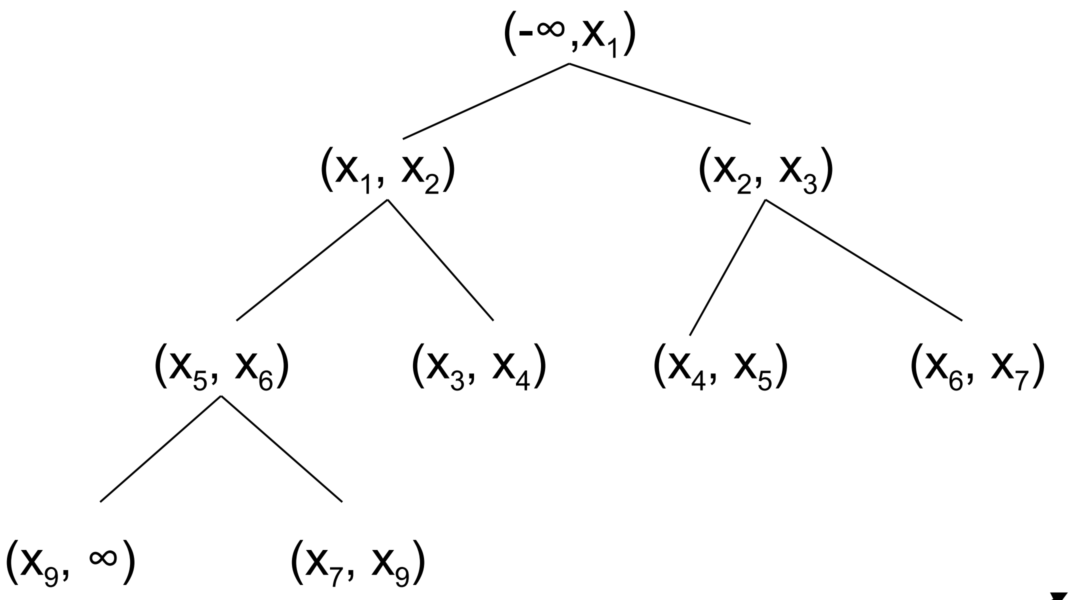
How to delete elements?



How to delete elements?



How to compute the accumulated value?



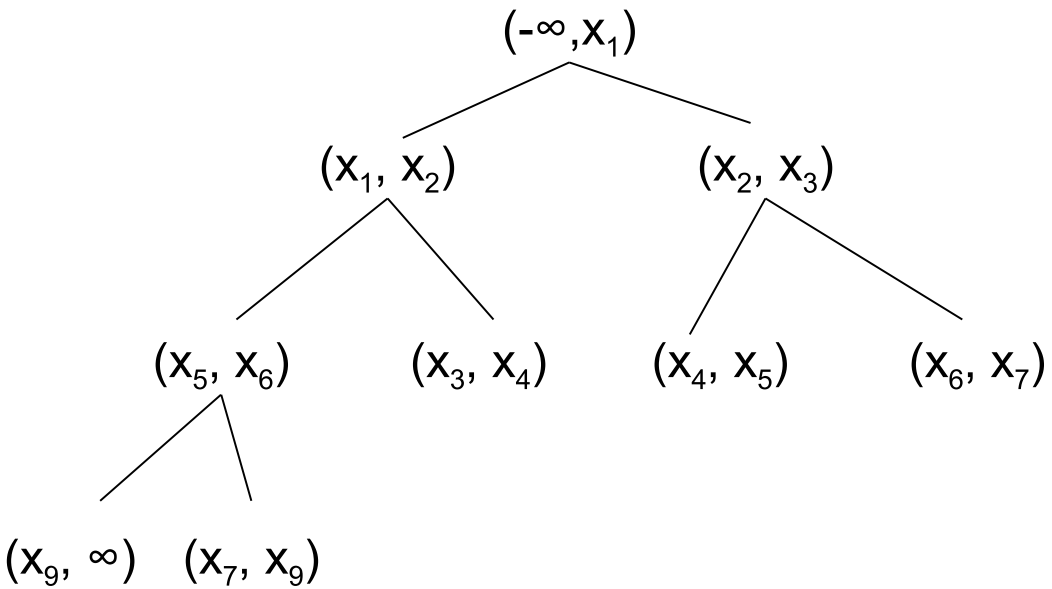
A pair (x_i, x_j)

$Proof_N = H(Proof_{left} || Proof_{right} || value)$

$Proof_{Nil} =$ “”

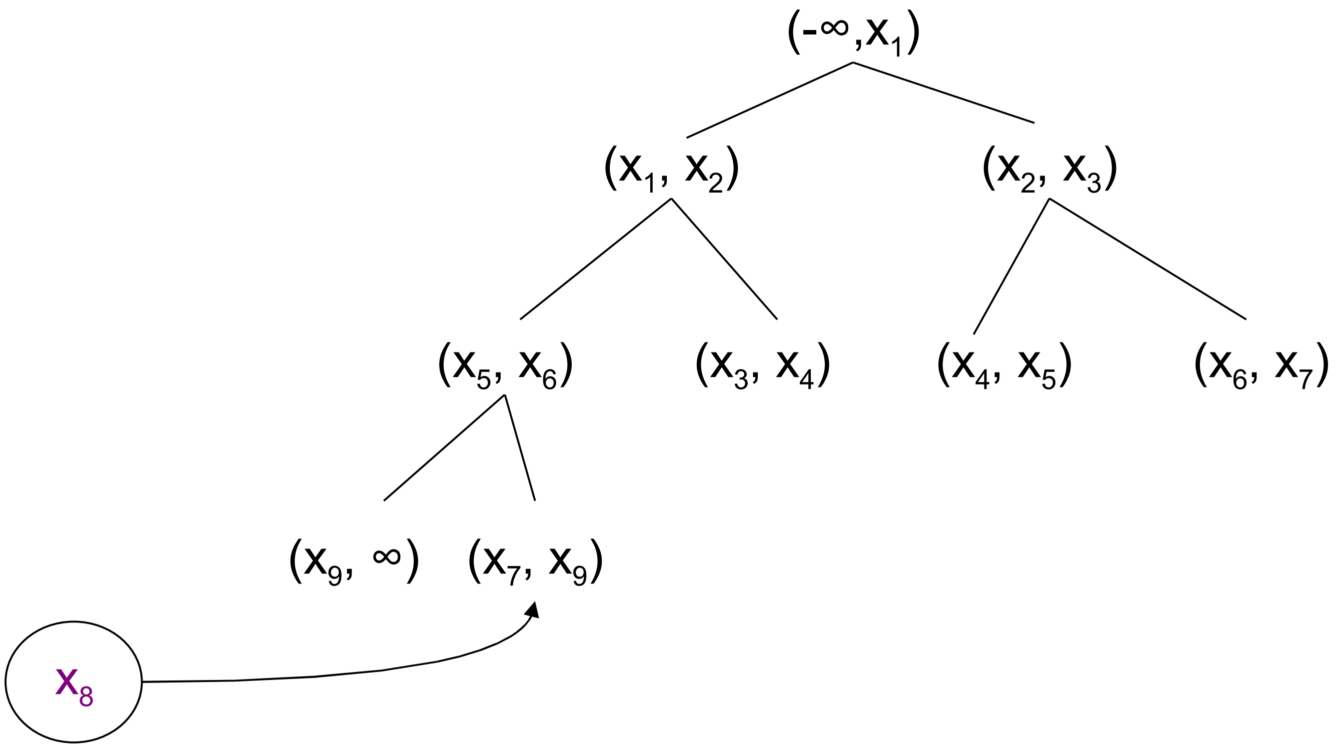
$Acc = Proof_{Root}$

How to update the accumulated value? (Insertion)



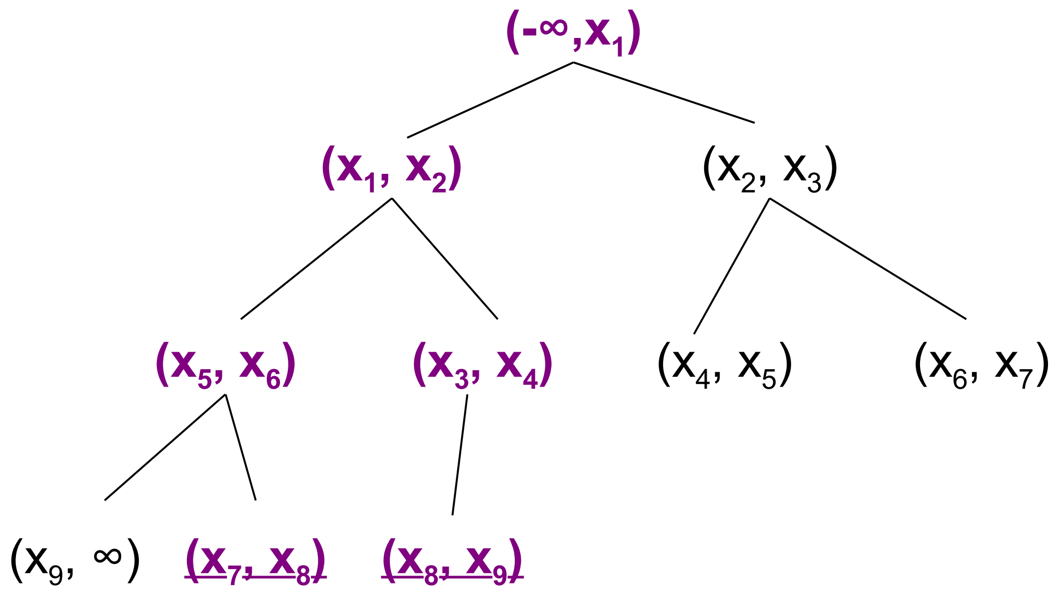
x_8 to be inserted.

How to update the accumulated value? (Insertion)



We will need to recompute proof node values.

How to update the accumulated value? (Insertion)



New element: x_8.

$Proof_N$ stored in each node.

Dark nodes do not require recomputing $Proof_N$.

Only a logarithmic number of values need recomputation.

Security

- **Definition:** an accumulated value Acc represents the set $X=\{x_1,x_2,\ldots,x_n\}$, if it has been computed from a tree T containing node values $\{(-\infty,x_1),(x_1,x_2),\ldots,(x_n,\infty)\}$, where each pair appears only once.

Security

- **Definition:** (Consistency)
  - Given Acc that represents X, it is hard to find witnesses that allow to prove inconsistent statements.
    - X={1,2}.
    - Hard to compute a *membership* witness for 3.
    - Hard to compute a *nonmembership* witness for 2.

Security

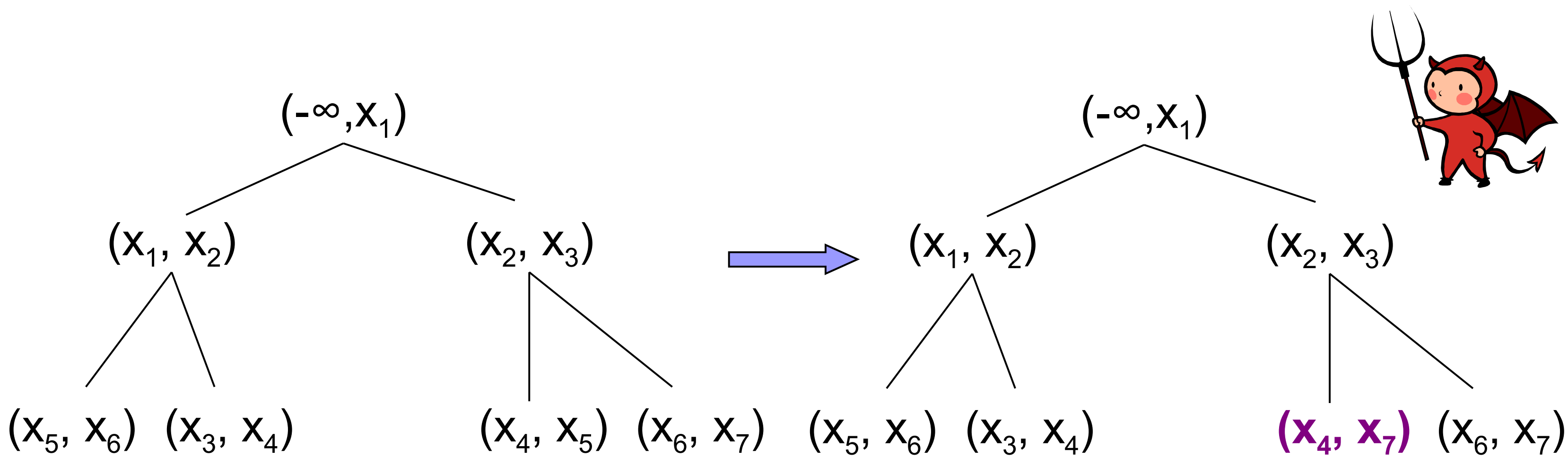
- **Definition:** (Update)
  - Guarantees that the accumulated value Acc represents the set X after insertion/deletion of x.
  - Every update must be checked by users but it is not needed to store the sequence of insertion/deletion.

Security

- **Theorem:** if CRHF exist the accumulator is secure (i.e. satisfies consistency and update).

Security

- **Lemma:** Given a tree T with accumulated value $Proof_T$, finding a tree T', $T \neq T'$ such that $Proof_T = Proof_{T'}$ is difficult.
- *Proof (Sketch):* $Proof_N = H(Proof_{left} || Proof_{right} || value)$



Security

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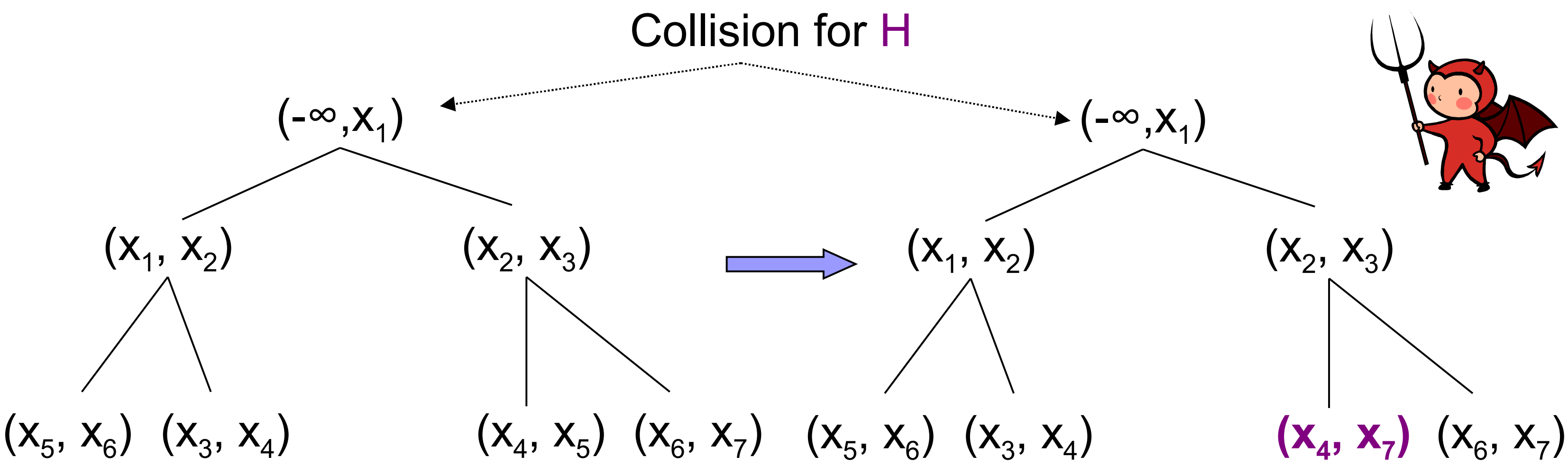

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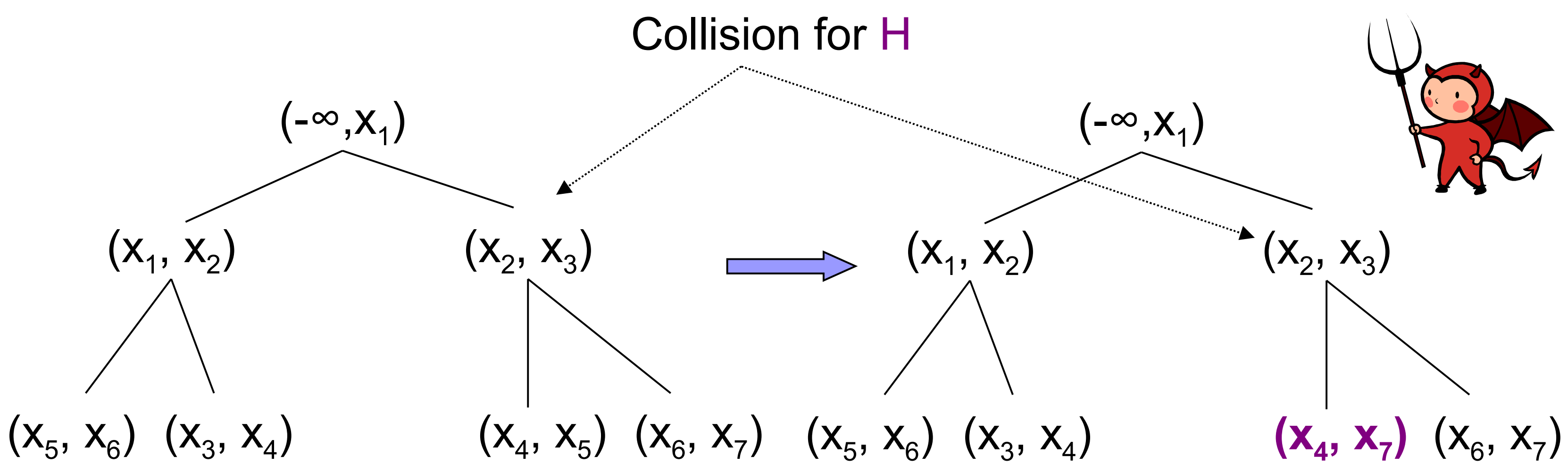
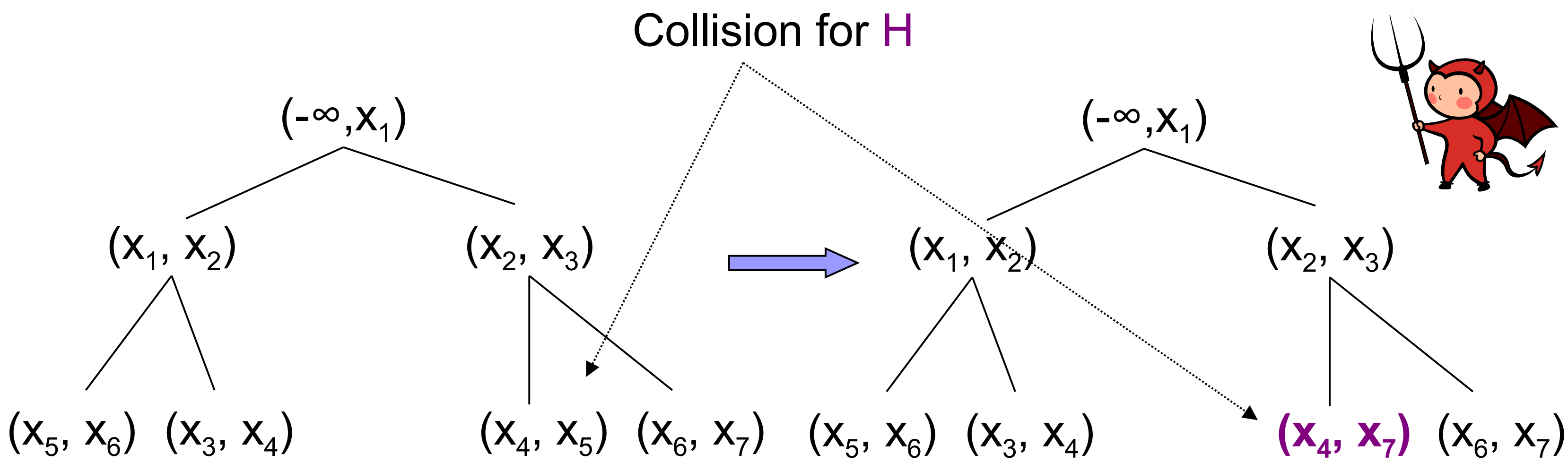
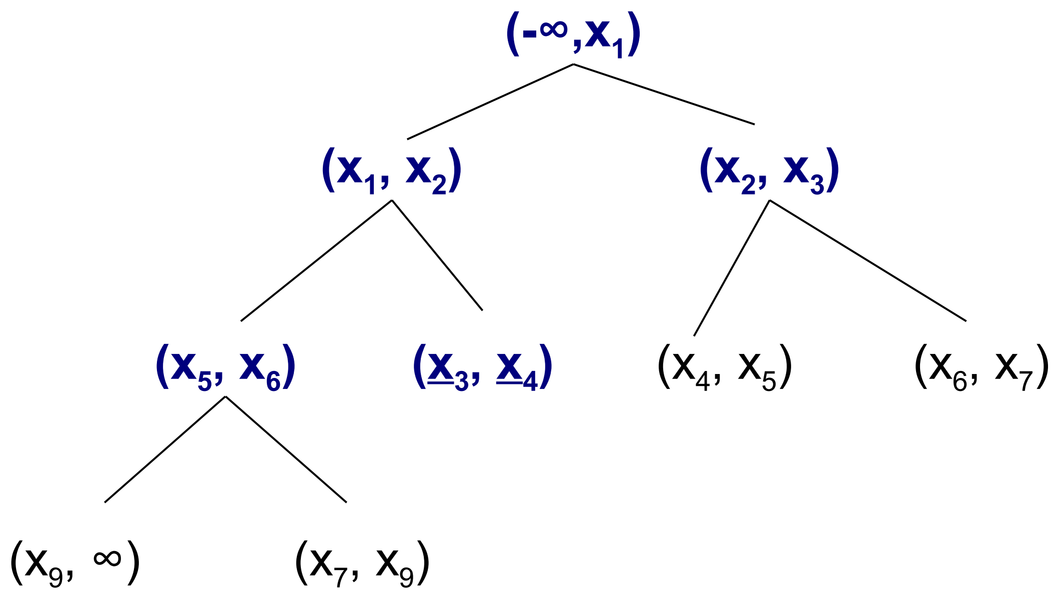


Security

- **Lemma:** Given a tree T with accumulated value $Proof_T$, finding a tree T', T≠T' such that $Proof_T = Proof_{T'}$ is difficult.
- *Proof (Sketch):* $Proof_N = H(Proof_{left}||Proof_{right}||value)$



Security (Consistency)



Witness: blue nodes and the **(x_3,x_4)** pair, size in $O(\ln(|X|))$

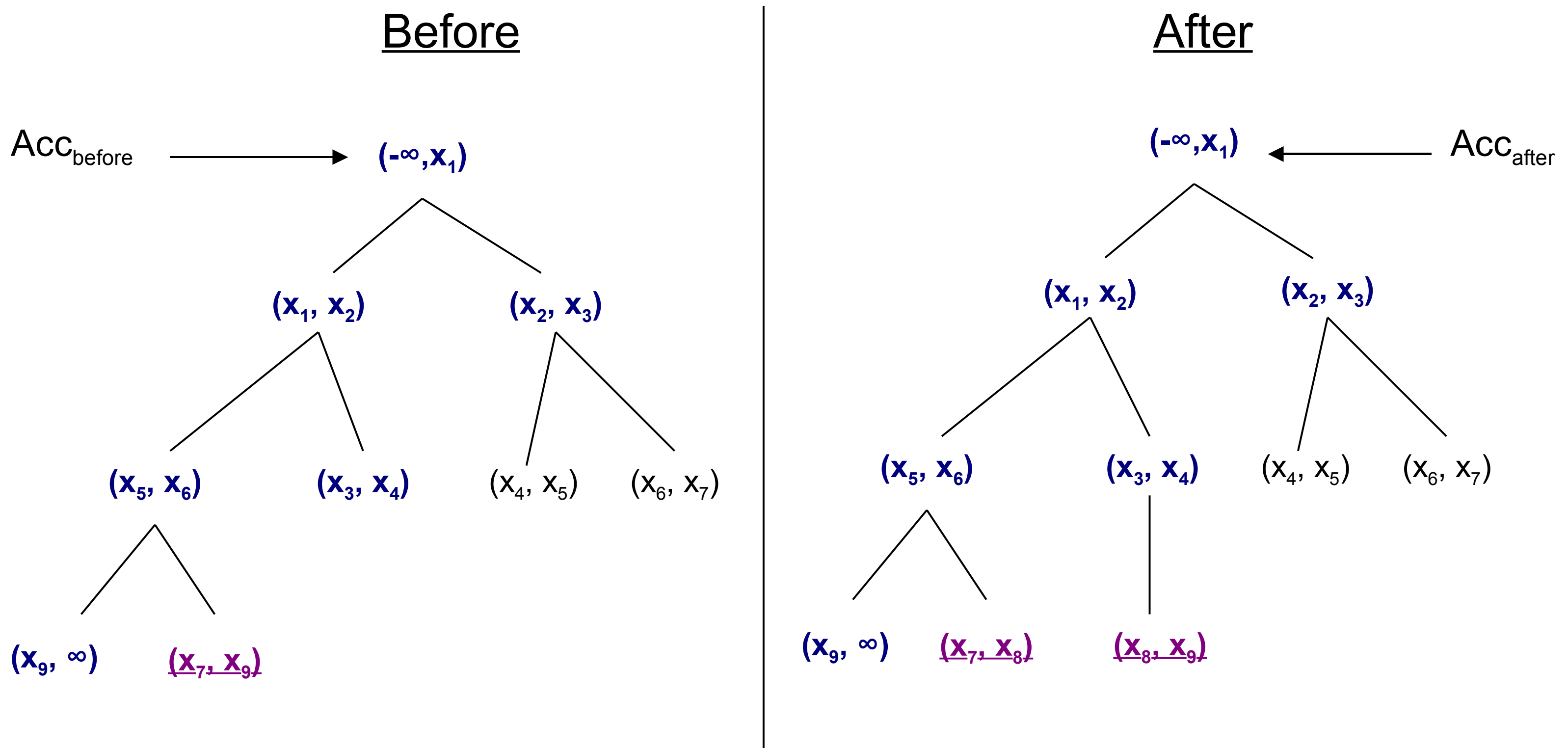
Checking that x belongs (or not) to X:

1) compute recursively the proof P and verify that P=Acc

2) check that: $x=x_3$ or $x=x_4$ (membership)

$x_3 < x < x_4$ (nonmembership)

Security (Update)



Insertion of x_8

Conclusion & Open Problem

- First *dynamic, universal, strong* accumulator
- Simple
- Security
  - Existence of CRHF
- Solves the e-Invoice Factoring Problem
- Less efficient than other constructions
  - Size of witness in O(ln(|X|))
- Open Problems
  - Is it possible to build an efficient *strong*,*dynamic* and *universal* accumulator with witness size lower than O(ln(|X|))?
  - How to handle more complex queries than membership? For example range queries, pattern queries on binary strings.

Thank you!

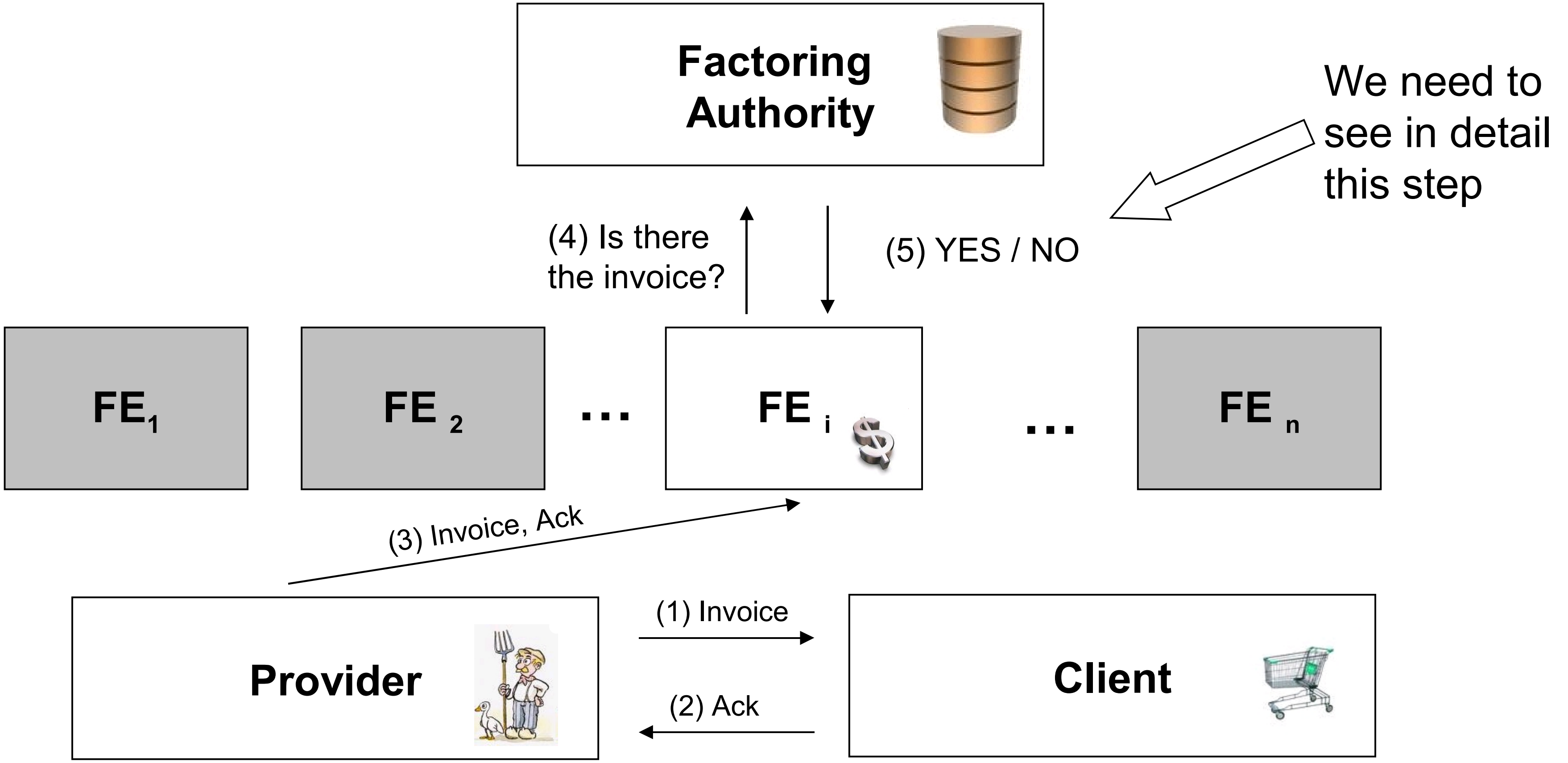
Distributed solutions?

- Complex to implement
- Hard to make them robust
- High bandwith communication
- Need to be online – synchronization problems
- **That’s why we focus on a centralized solution.**

Invoice Factoring using accumulator

- We need a secure broadcast channel
  - If a message m is published, every participant sees the same m.
- Depending on the security level required
  - Trusted http of ftp server
  - Bulletin Board [CGS97]

Invoice Factoring using accumulator



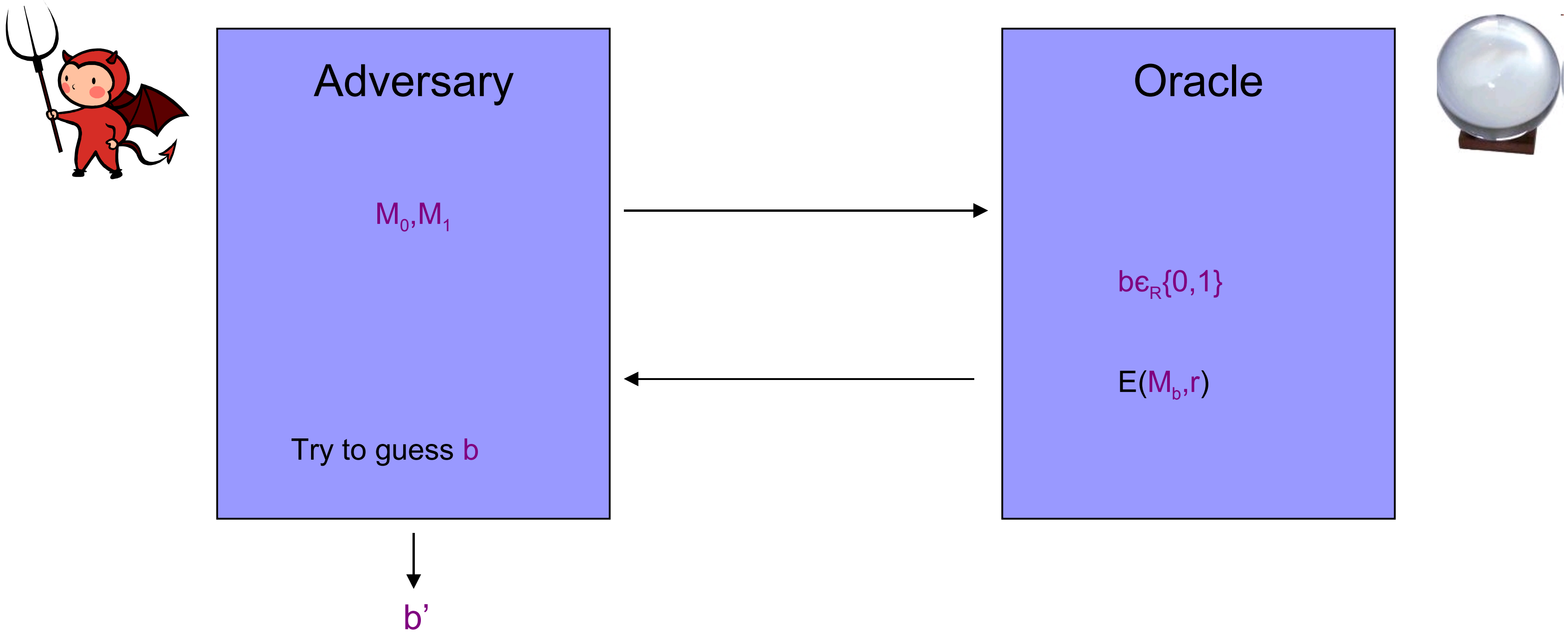
Invoice Factoring using accumulator

- Step 5 (Details)

FE		Factoring Authority
	Have you got invoice x? →	
	← YES/NO, w	$w = Witness(M_{before}, x)$
$Belongs(x, w, Acc_{before})$	If NO, insert x →	
	← $Acc_{after}, w_{up}, ID_{FE}$	$Acc_{after}, M_{after}, w_{up} = Update_{Add}(M_{before}, x)$
$CheckUpdate(Acc_{before}, Acc_{after}, w_{up})$		
All tests pass => I can buy x.		

Basic Cryptographic Notions

- Secure encryption [GM84]



Adversary wins if **$\Pr[b=b'] > \frac{1}{2} + 1/q(n)$**

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