DEPARTAMENTO DE CIENCIAS DE LA COMPUTACIÓN
UNIVERSIDAD DE CHILE

latin crypt
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On the Impossibility of Batch Update for Cryptographic Accumulators

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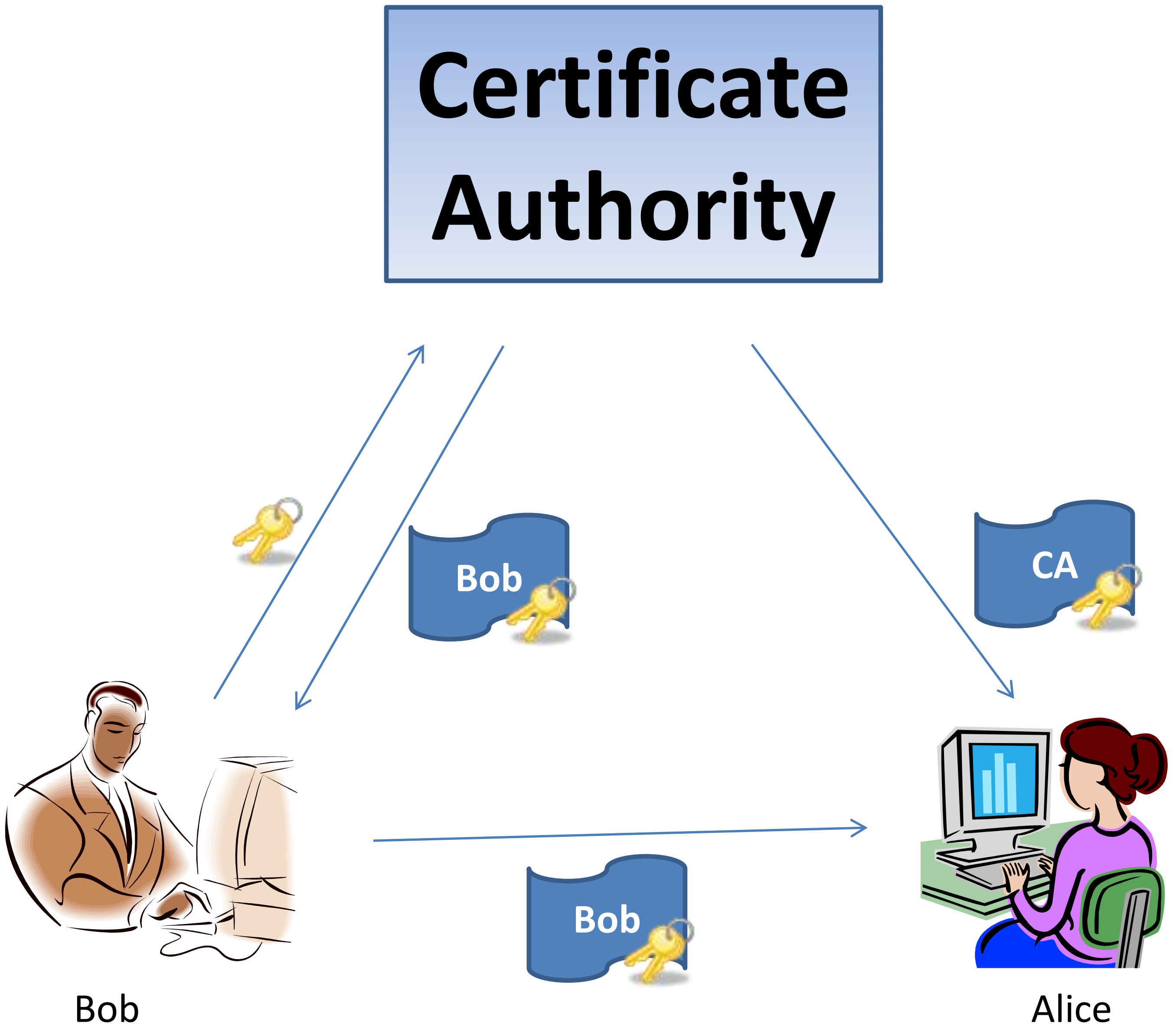
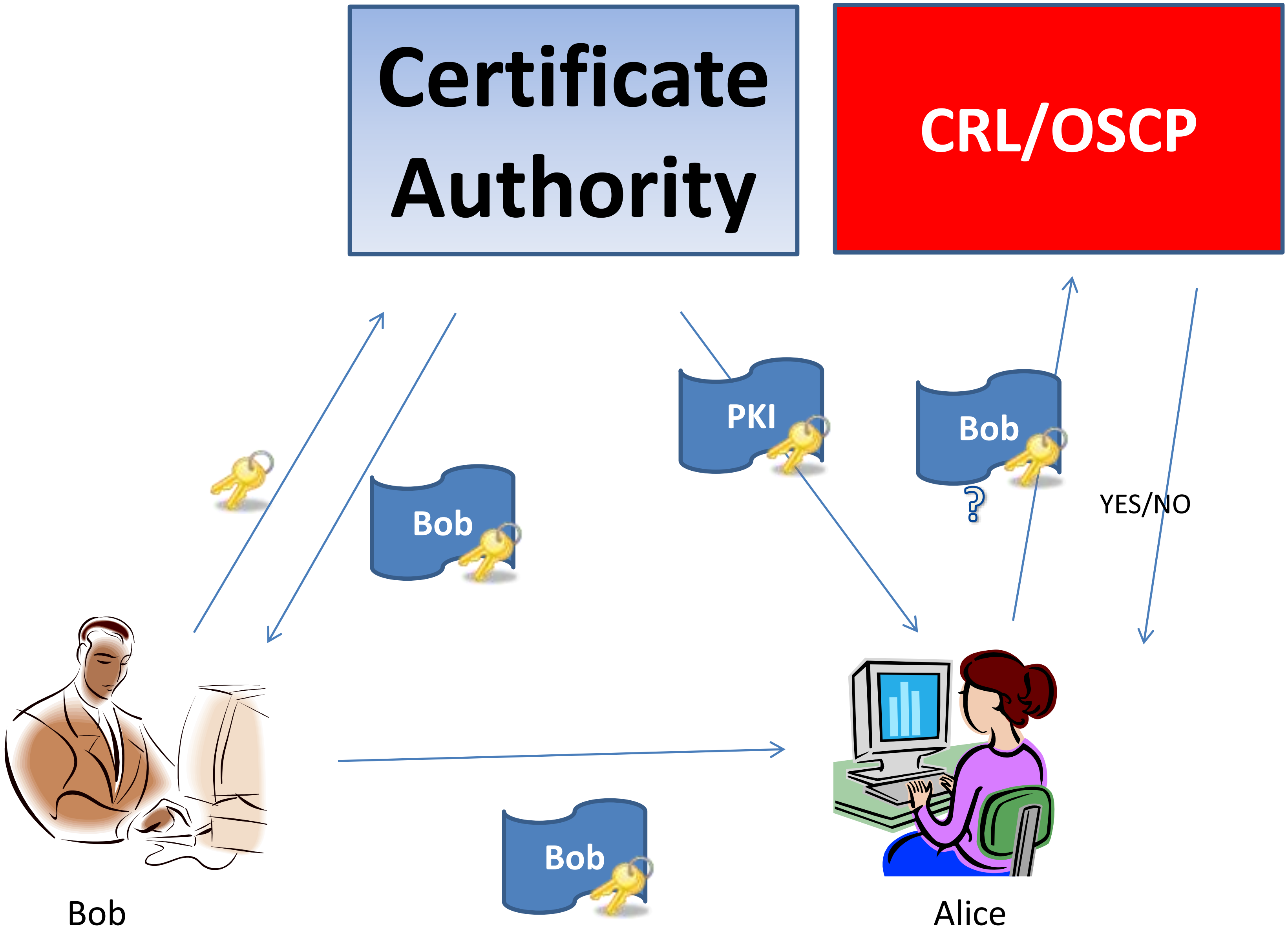
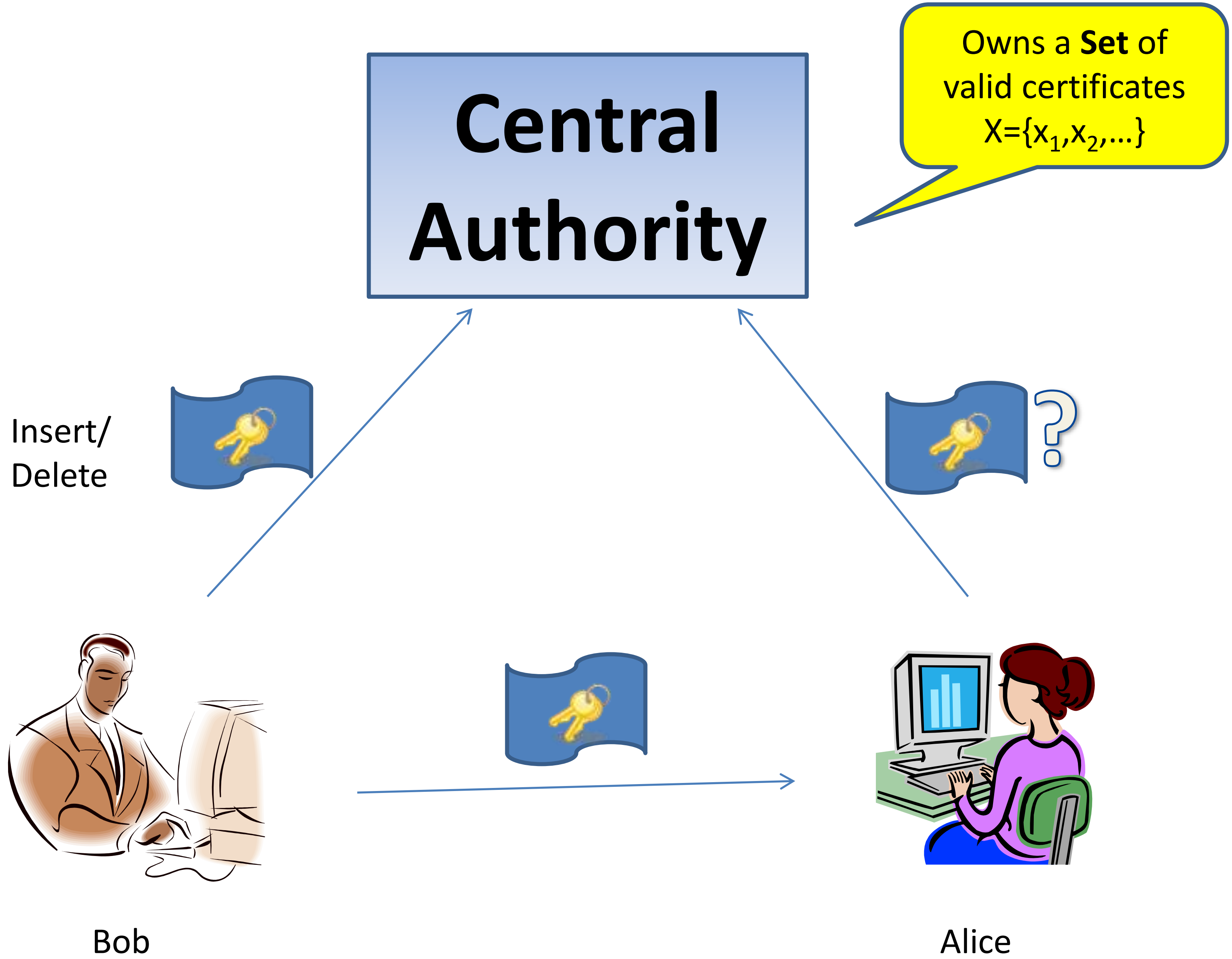
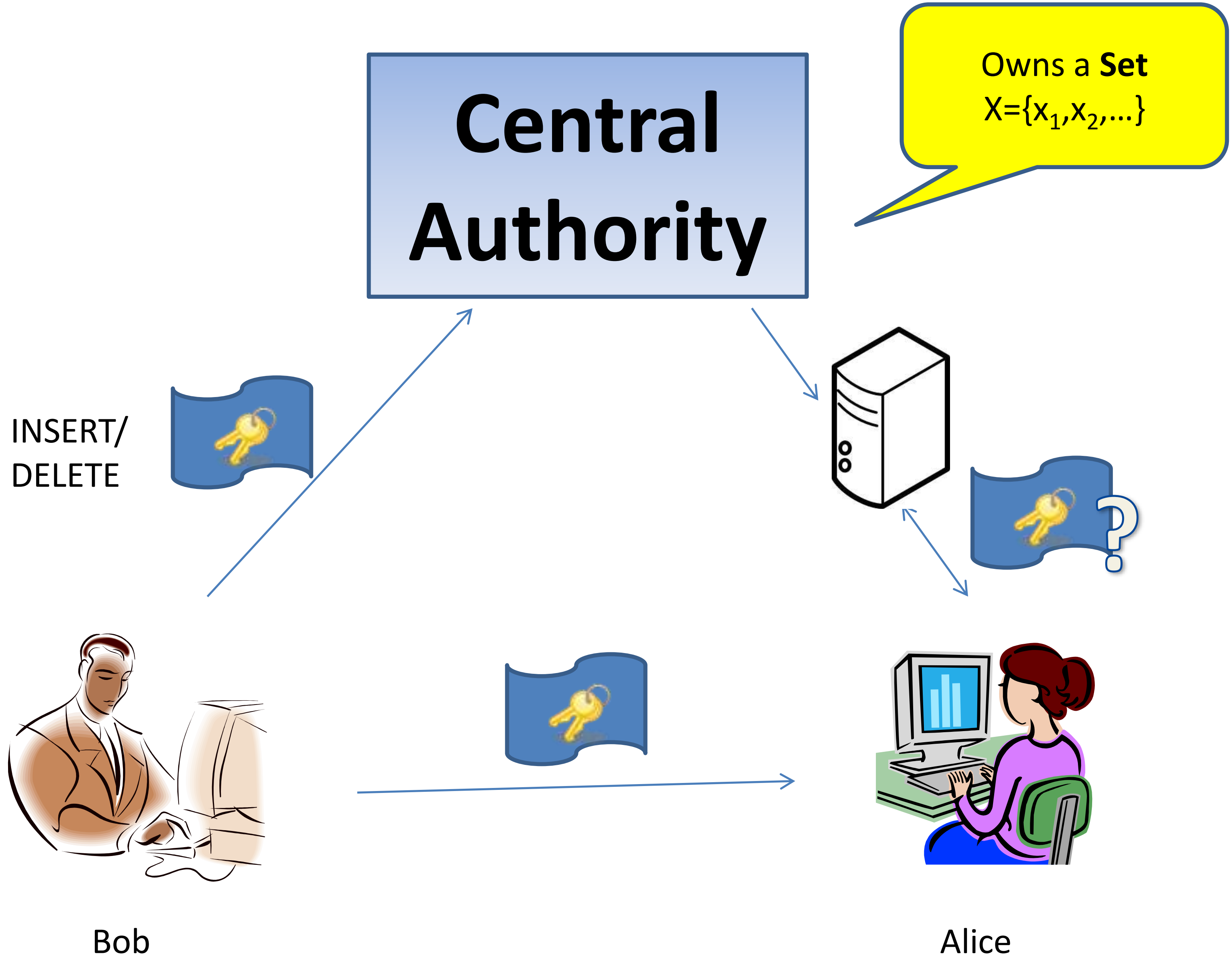
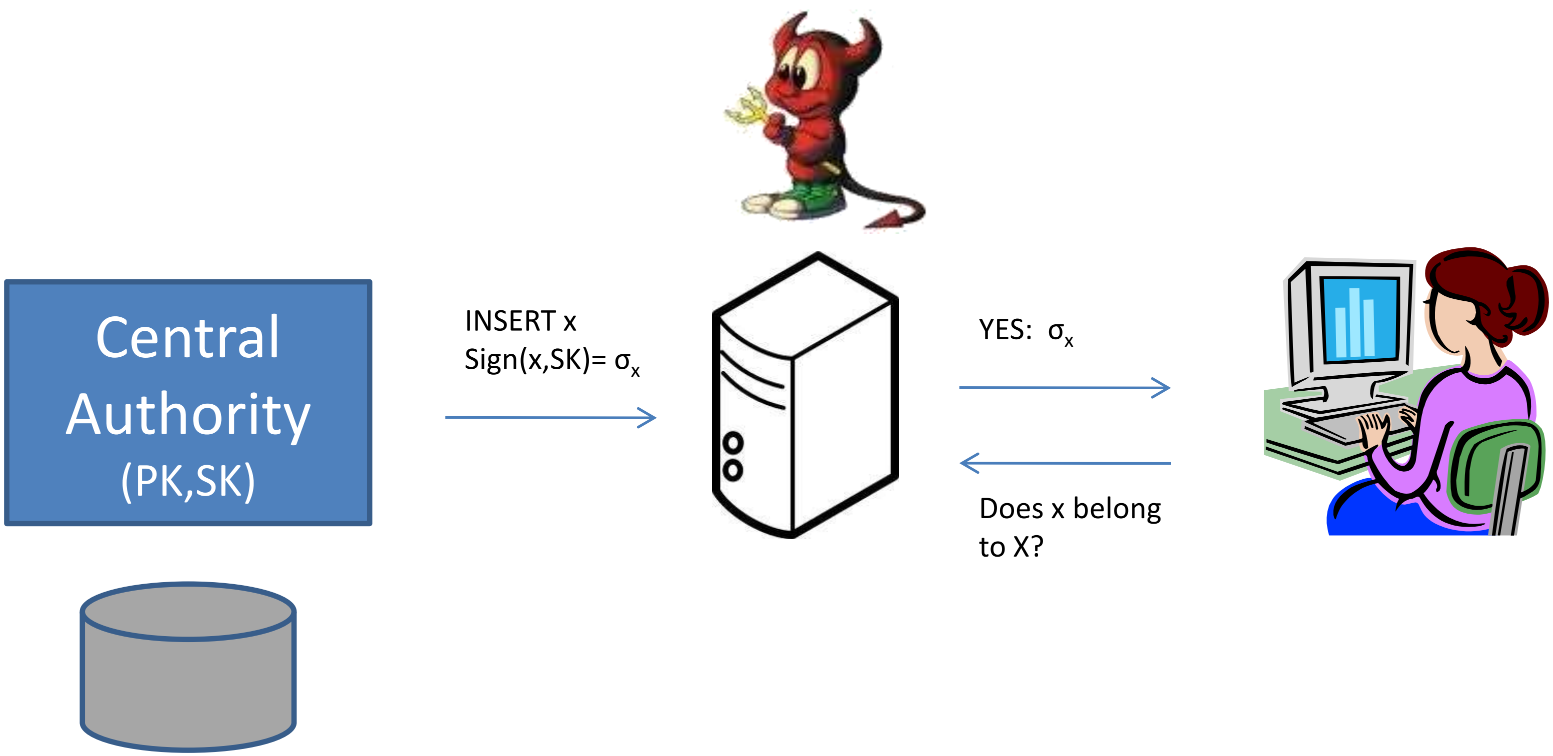

Certificate Authority
Bob
CA
Bob
Bob
Alice


Certificate Authority
CRL/OSCP
PKI
Bob
Bob
?
YES/NO
Bob
Bob
Alice

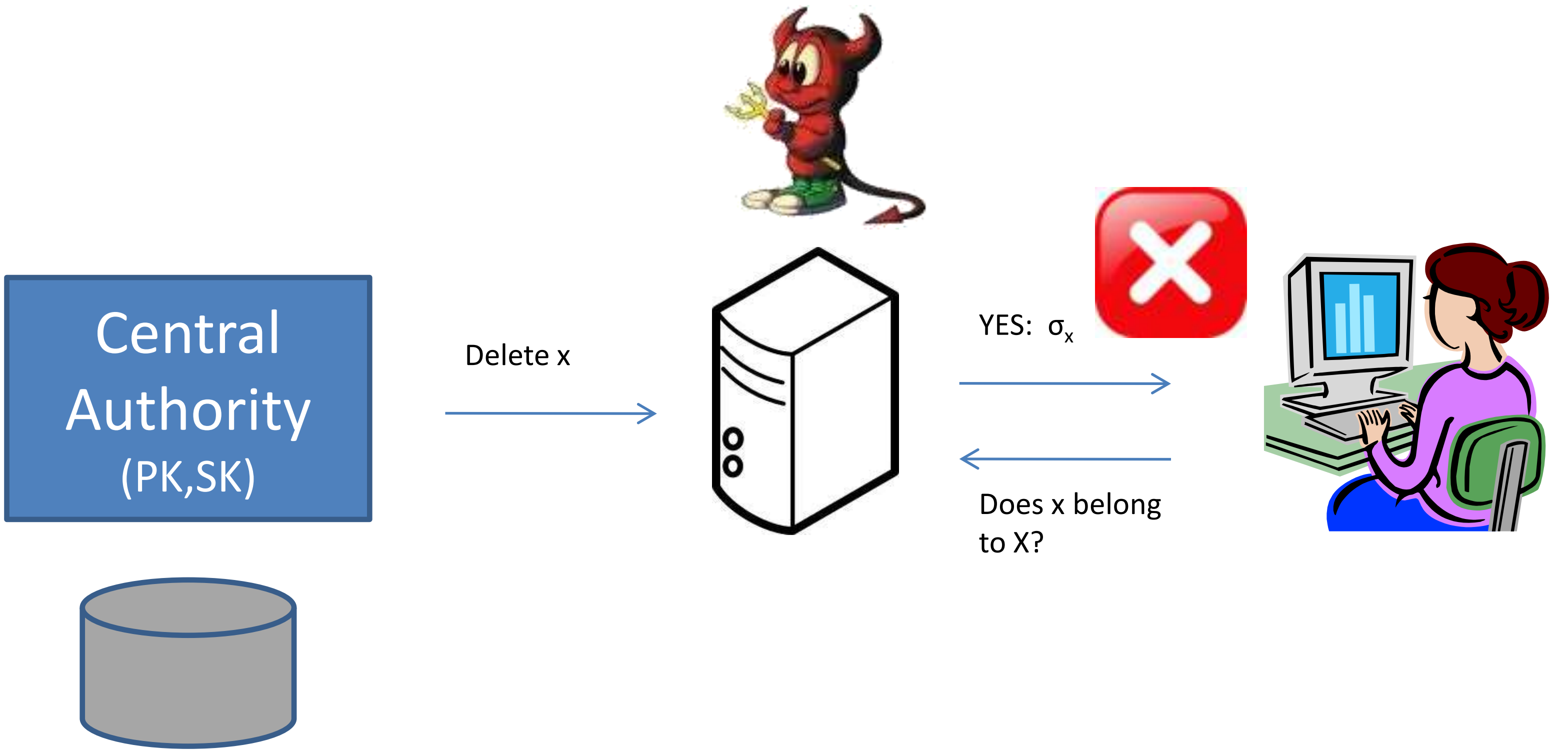

Central Authority
Owns a **Set** of valid certificates $X=\{x_1,x_2,\ldots\}$
Insert/ Delete
?
Bob
Alice

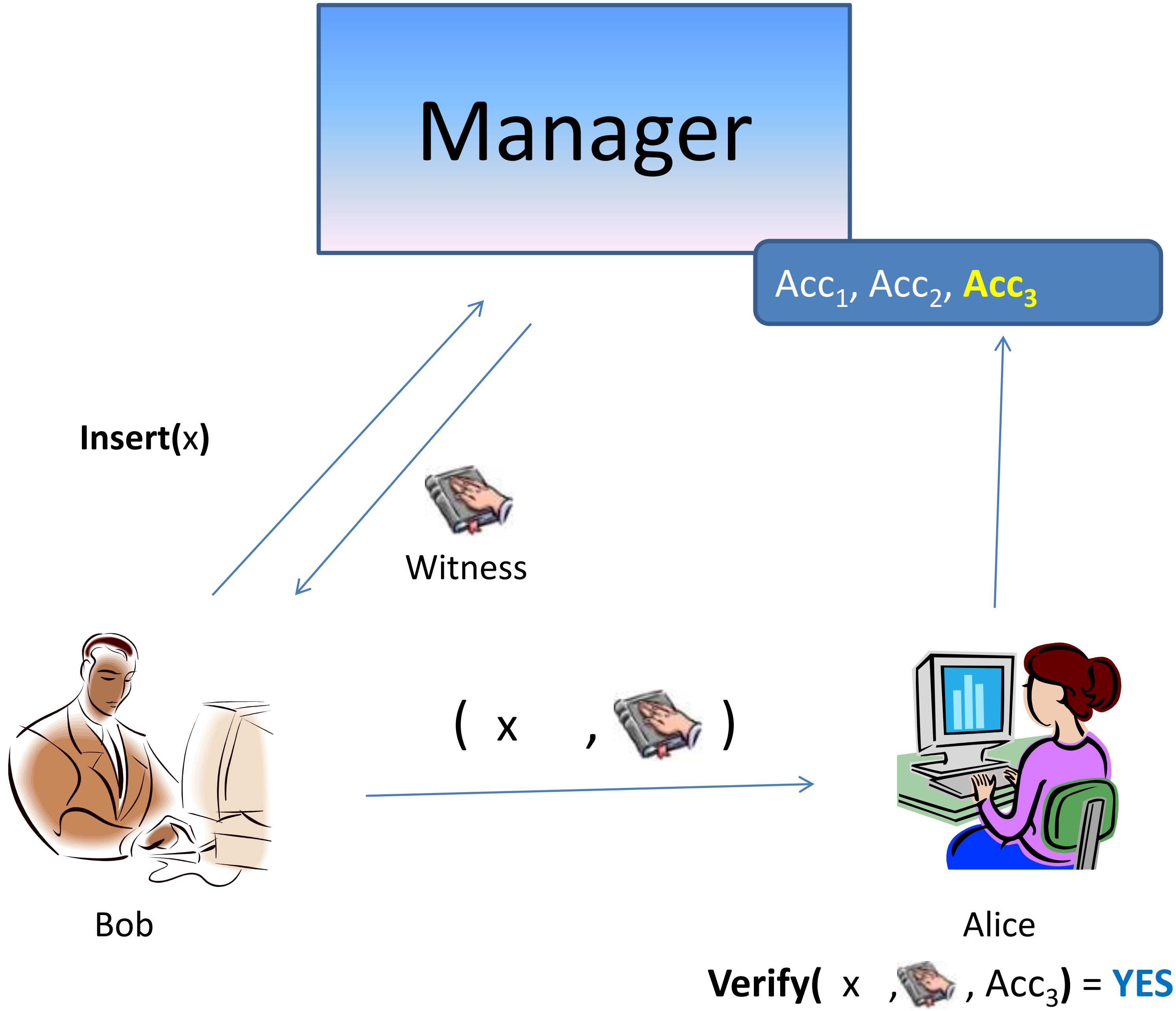

Central Authority
Owns a **Set** $X=\{x_1,x_2,\ldots\}$
INSERT/
DELETE
Bob
Alice

Replay Attack

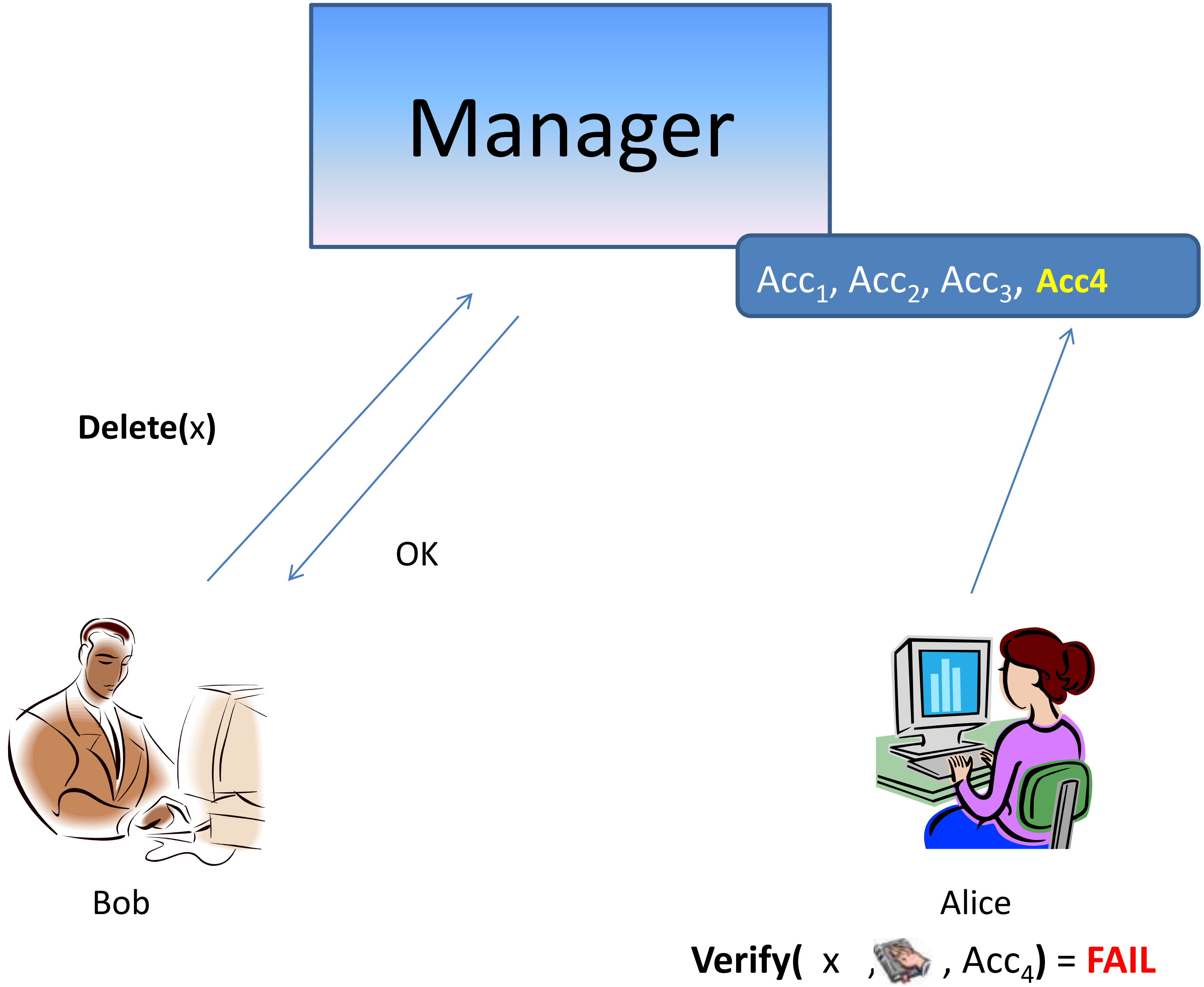


Replay Attack



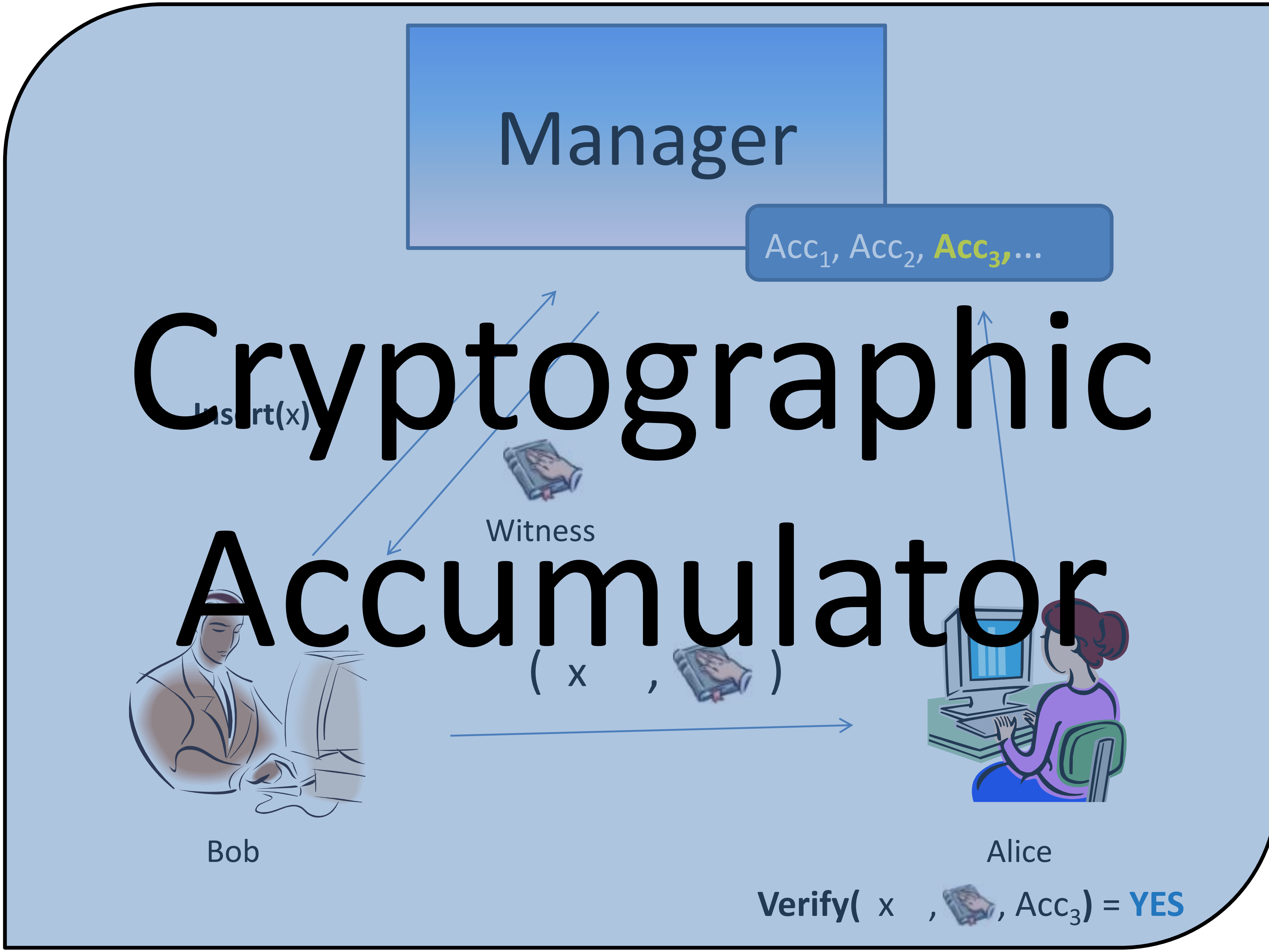


Manager
Acc_1, Acc_2, Acc_3
Insert(x)
Witness
(x ,)
Bob
Alice
Verify(x , , Acc_3) = YES



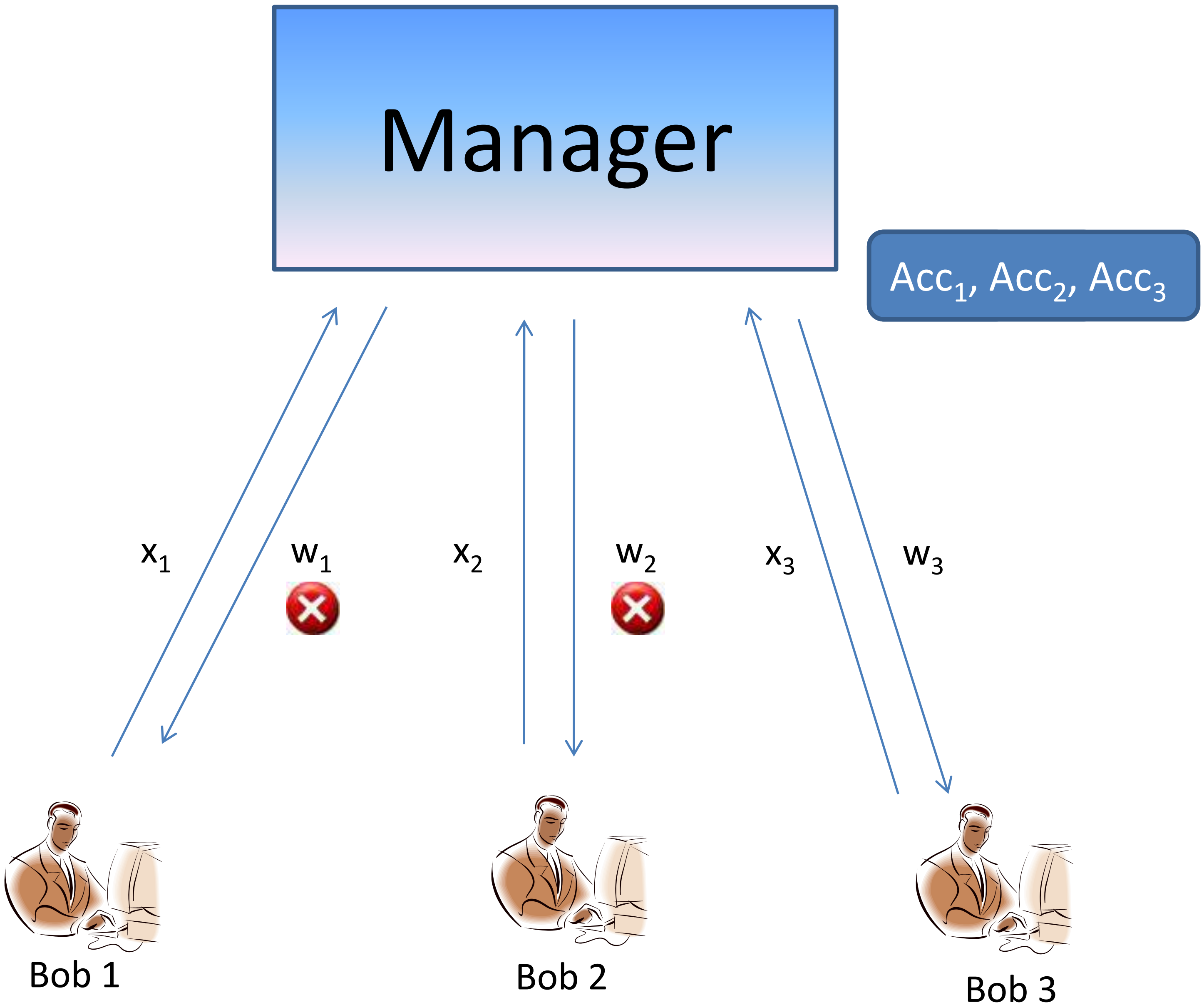
Manager
Acc1, Acc2, Acc3, Acc4
Delete(x)
OK
Bob
Alice
Verify(x , , Acc4) = FAIL

Cryptographic Accumulator



Main constructions

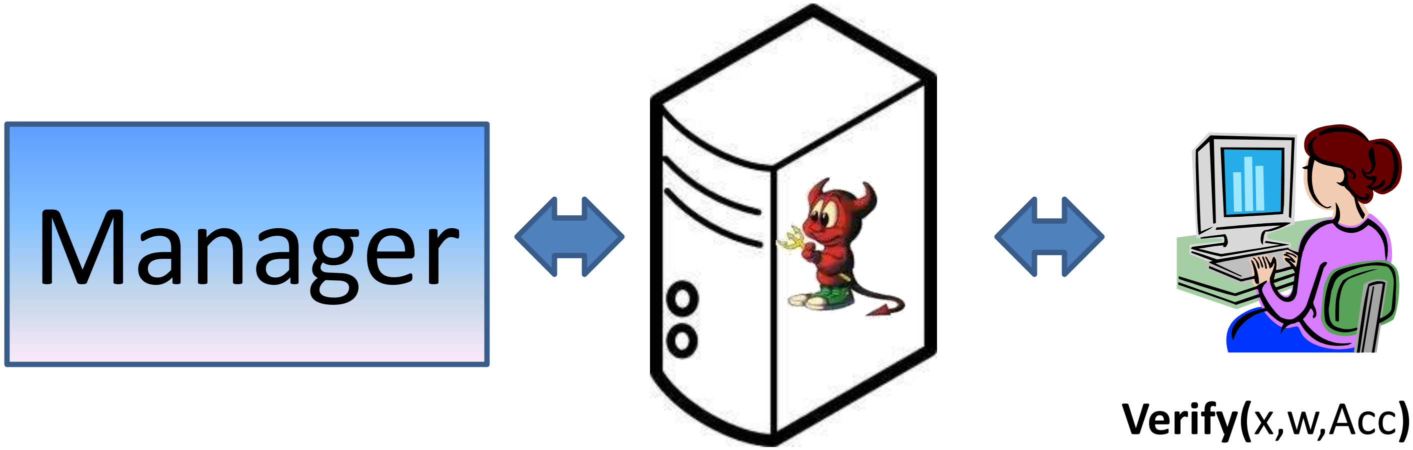
	Security	Note
[BeMa94]	RSA + RO	First definition
[BarPfi97]	Strong RSA	-
[CamLys02]	Strong RSA	First dynamic accumulator
[LLX07]	Strong RSA	First universal accumultor
[Ngu05]	Pairings	E-cash, ZK-Sets,…
[WWP08]	eStrong RSA Paillier	Batch Update
[CHKO08]	Collision-Resistant Hashing	Untrusted Manager
[CKS09]	Pairings	Group multiplication



Manager
Acc1, Acc2, Acc3
x1
w1
x2
w2
x3
w3
Bob 1
Bob 2
Bob 3

Problem: after each update of the accumulated value it is necesarry to recompute all the witnesses.

Delegate Witness Computation?



Constructions	Replica (Compute a single witness)	User (Verify)
[CL02]	O(|X|)	O(1)
[GTT09]	$O(\|X\|^{1/\varepsilon})$	O(ε)
[CHK08]	O(log |X|)	O(log |X|)

Batch Update [FN02]

Manager

…, Acc_{99}, $\mathbf{Acc_{100}}$, Acc_{101}, …, $\mathbf{Acc_{200}}$, …

$Upd_{100,200}$

Bob 1

(x_1, w_1, Acc_{100})
(x_2, w_2, Acc_{100})
(x_6, w_6, Acc_{100})

Bob 2

$(x_{36}, w_{36}, Acc_{100})$
$(x_{87}, w_{87}, Acc_{100})$

Bob 29

(x_1, w_1, Acc_{100})
$(x_{20}, w_{20}, Acc_{100})$
$(x_{69}, w_{68}, Acc_{100})$
$(x_{64}, w_{64}, Acc_{100})$
…

Bob 42

(x_1, w_1, Acc_{100})
(x_2, w_2, Acc_{100})
(x_6, w_6, Acc_{100})
….

Batch Update [FN02]

Manager

$\ldots, Acc_{99}$, $\mathbf{Acc_{100}}$, $Acc_{101}, \ldots$, $\mathbf{Acc_{200}}, \ldots$

Bob 1

$(x_1, \mathbf{w_1', Acc_{200}})$
$(x_2, \mathbf{w_2', Acc_{200}})$
$(x_6, \mathbf{w_6', Acc_{200}})$

Bob 2

$(x_{36}, \mathbf{w_{36}', Acc_{200}})$
$(x_{87}, \mathbf{w_{87}', Acc_{200}})$

Bob 29

$(x_1, \mathbf{w_1', Acc_{200}})$
$(x_{20}, \mathbf{w_{20}', Acc_{200}})$
$(x_{69}, \mathbf{w_{68}', Acc_{200}})$
$(x_{64}, \mathbf{w_{64}', Acc_{200}})$
…

Bob 42

$(x_1, \mathbf{w_1', Acc_{200}})$
$(x_2, \mathbf{w_2', Acc_{200}})$
$(x_6, \mathbf{w_6', Acc_{200}})$
….

Batch Update [FN02]

Trivial solution:

Upd_{X_i,X_j} = {list of all witnesses for X_j}

More interesting:

$|Upd_{X_i,X_j}| = O(1)$

What happens with [CL02]?

- PK=(n,g) with n=pq and g ϵ $\mathbf{Z}_n^*$
- $Acc_{\emptyset}$:= g mod n
- **Insert(**x,Acc**)** := Acc^x mod n /* x prime */
- **Delete(**x,Acc**)** := $Acc^{1/x}$ mod n
- **WitGen(**x,Acc**)** $\stackrel{?}{:=}$ $Acc^{1/x}$ mod n
- **Verify(**x,w,Acc**)**: $w^x = Acc$
- $|\mathbf{Upd}_{x_i,x_j}|$ = **O(|{list of insertions / deletions}|)**

Syntax of B.U. Accumulators

Algorithm	Returns	Who runs it
KeyGen(1^k**)**	**PK,SK,**$\mathbf{Acc}_{\emptyset}$	Manager
AddEle(x,Acc_X,SK**)**	$\mathbf{Acc}_{X \cup \{x\}}$	Manager
DelEle(x,Acc_X,SK**)**	$\mathbf{Acc}_{X \setminus \{x\}}$	Manager
WitGen(x,Acc_X,SK**)**	Witness **w** relative to $\mathbf{Acc}_X$	Manager
Verify(x,w,Acc_X,PK**)**	Returns **Yes** whether $\mathbf{x \in X}$	User
UpdWitGen(X,X',SK**)**	$\mathbf{Upd}_{X,X'}$ for elements $\mathbf{x \in X \cap X'}$	Manager
UpdWit(w,Acc_X,$Acc_{X'}$,$Upd_{X,X'}$,PK**)**	New witness **w'** for $\mathbf{x \in X'}$	User

Correctness

- **Definition**

The scheme is correct iff:

w := **WitGen(**x, Acc_X, SK**)** $\Rightarrow$ **Verify(**x, w, Acc_X, PK**)** = Yes

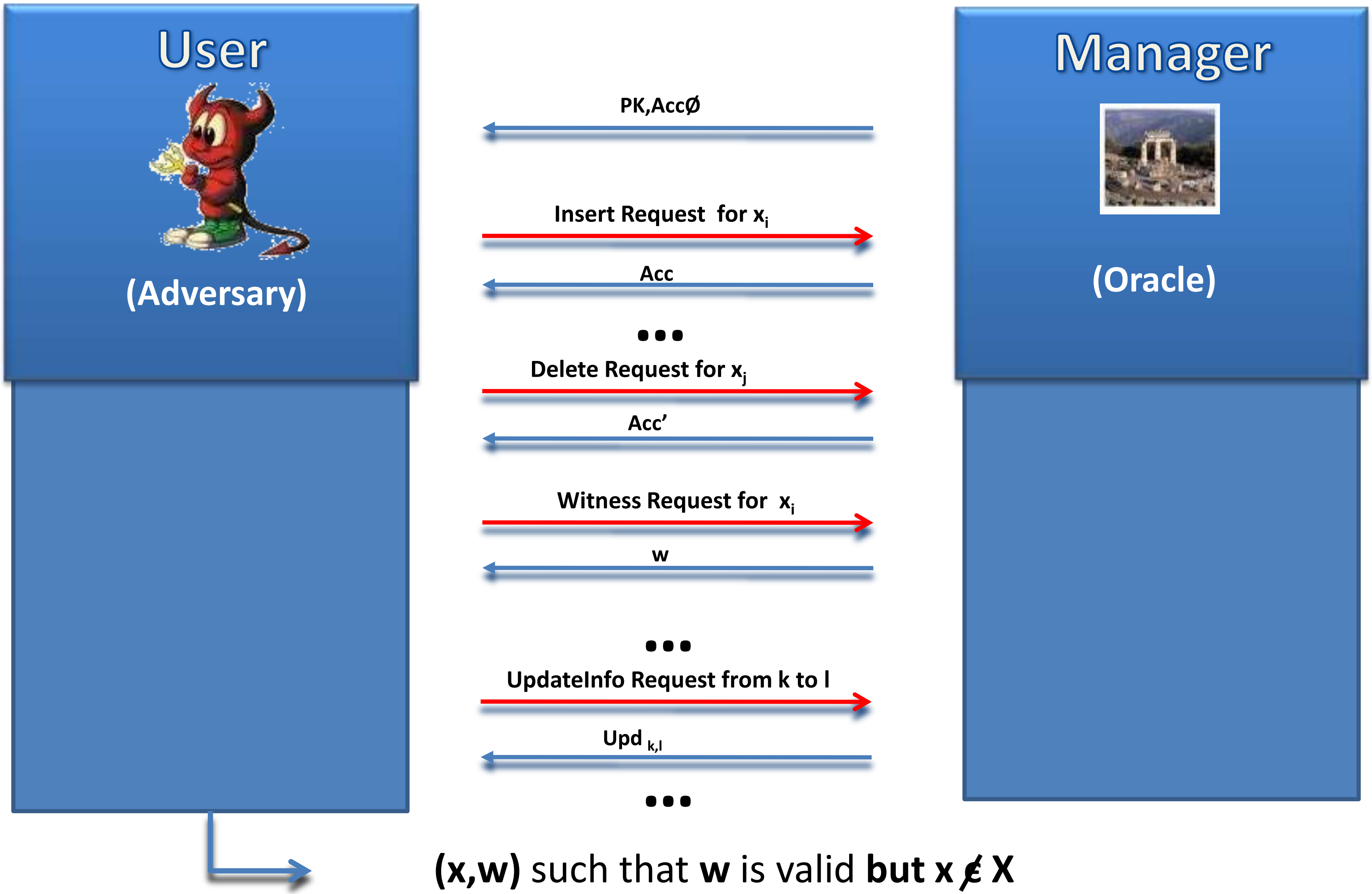
w := **WitGen(**x, Acc_X, SK**)**

$Upd_{X,X'}$:= **UpdWitGen(**X, X', SK**)**

w' := **WitGen(**$w, Acc_X, Acc_{X'}, Upd_{X,X'}, PK$**)**

$\Rightarrow$ **Verify(**$x, w', Acc_{X'}, PK$**)** = Yes

Security Model [CL02,WWP08]



Batch Update Construction [WWP08]

CONSTRUCTION. Wang et al.'s accumulator relies on the Paillier cryptosystem [8] which we recall in Appendix A.2. In the following, λ will denote the value $lcm(p-1, q-1)$ where $n = pq$ is a product of large-enough safe primes p, q, and $F : u \to \frac{u-1}{n}$ is Paillier's L function [8].

- KeyGen(1^k): given the security parameter k in unary, compute a safe-prime product $n = pq$ that is k-bits long and create an empty set V. Let $\mathcal{C} = \mathbb{Z}^*_{n^2} \setminus \{1\}$ and $T' = \{3, ..., n^2\}$. Let $\beta \xleftarrow{R} \mathbb{Z}^*_{\varphi(n^2)}$ and $\sigma \xleftarrow{R} \mathbb{Z}^+$ be two random numbers. The public key PK is set to (n, β) and the private key SK to (σ, λ). The output is the parameter $\mathcal{P} = (PK, SK)$.
- AccVal($X, \mathcal{P}$): given a set $X = \{c_1, ..., c_m\}$ with $X \subset \mathcal{C}$, and the parameter $\mathcal{P}$, take $c_{m+1} \xleftarrow{R} \mathcal{C}$ and compute

$$\begin{aligned} x_i &= F(c_i^\lambda \bmod n^2) \bmod n \ \ (\text{for } i = 1, ..., m+1) \\ Acc_X &= \sigma \textstyle\sum_{i=1}^{m+1} x_i \bmod n \\ y_i &= c_i^{\lambda\sigma\beta^{-1}} \bmod n^2 \ \ (\text{for } i = 1, ..., m+1) \\ a_c &= \Pi_{i=1}^{m+1} y_i \bmod n^2 \end{aligned}$$

  Output the accumulated value Acc_X and the auxiliary information a_c.
- WitGen($a_c, X, \mathcal{P}$): given the auxiliary information a_c, a set $X = \{c_1, ..., c_m\}$, and the parameter $\mathcal{P}$, choose uniformly at random a set of m numbers $T = \{t_1, ..., t_m\} \subset T' \setminus \{\beta\}$ (for $i = 1, ..., m$) and compute

$$w_i = a_c c_i^{-t_i\beta^{-1}} \bmod n^2 \ \ (\text{for } i = 1, ..., m)$$

  Output the witness $W_i = (w_i, t_i)$ for c_i (for $i = 1, ..., m$).
- AddEle($X^\oplus, a_c, Acc_X, \mathcal{P}$): given a set $X^\oplus = \{c_1^\oplus, ..., c_l^\oplus\} (X^\oplus \subseteq \mathcal{C} \setminus X)$, to be inserted, the auxiliary information a_c, the accumulated value Acc_X, and the parameter $\mathcal{P}$, choose $c_{l+1}^\oplus \xleftarrow{R} \mathcal{C}$ and a set of l numbers $T^\oplus = \{t_1^\oplus, ..., t_l^\oplus\} \xleftarrow{R} T' \setminus (T \cup \{\beta\})$, and compute

$$\begin{aligned} x_i^\oplus &= F((c_i^\oplus)^\lambda \bmod n^2) \bmod n \ \ (\text{for } i = 1, ..., l+1) \\ Acc_{X \cup X^\oplus} &= Acc_X + \sigma \textstyle\sum_{i=1}^{l+1} x_i^\oplus \bmod n \\ y_i^\oplus &= (c_i^\oplus)^{\lambda\sigma\beta^{-1}} \bmod n^2 \ \ (\text{for } i = 1, ..., l+1) \\ a_u &= \Pi_{i=1}^{l+1} y_i^\oplus \bmod n^2 \\ w_i^\oplus &= a_c a_u (c_i^\oplus)^{-t_i^\oplus \beta^{-1}} \bmod n^2 \ \ (\text{for } i = 1, ..., l) \end{aligned}$$

  Set $a_c = a_c a_u \bmod n^2, T = T \cup T^\oplus$, and $V = V \cup \{a_u\}$. Then output the new accumulated value $Acc_{X \cup X^\oplus}$ corresponding to the set $X \cup X^\oplus$, the witness $W_i^\oplus = (w_i^\oplus, t_i^\oplus)$ for the new added elements $c_i^\oplus$ (for $i = 1, ..., l$), and the auxiliary information a_u and a_c.
- DelEle($X^\ominus, a_c, Acc_X, \mathcal{P}$): given a set $X^\ominus = \{c_1^\ominus, ..., c_l^\ominus\} (X^\ominus \subset X)$ to be deleted, the auxiliary information a_c, the accumulated value Acc_X, and the parameter $\mathcal{P}$, choose $c_{l+1}^\ominus \xleftarrow{R} \mathcal{C}$ and compute

$$\begin{aligned} x_i^\ominus &= F((c_i^\ominus)^\lambda \bmod n^2) \bmod n \ \ (\text{for } i = 1, ..., l+1) \\ Acc_{X \setminus X^\ominus} &= Acc_X - \sigma \textstyle\sum_{i=1}^{l} x_i^\ominus + \sigma x_{l+1}^\ominus \bmod n \\ y_i^\ominus &= (c_i^\ominus)^{\lambda\sigma\beta^{-1}} \bmod n^2 \ \ (\text{for } i = 1, ..., l+1) \\ a_u &= y_{l+1}^\ominus \Pi_{j=1}^{l} (y_j^\ominus)^{-1} \bmod n^2 \end{aligned}$$

  Set $a_c = a_c a_u \bmod n^2$ and $V = V \cup \{a_u\}$. Then output the new accumulated value $Acc_{X \setminus X^\ominus}$ corresponding to the set $X \setminus X^\ominus$ and the auxiliary information a_u and a_c.
- Verify(c, W, Acc_X, PK): given an element c, its witness $W = (w, t)$, the accumulated value Acc_X, and the public key PK, test whether $\{c, w\} \subset \mathcal{C}, t \in T'$ and $F(w^\beta c^t \bmod n^2) \equiv Acc_X(\bmod n)$. If so, output Yes, otherwise output $\perp$.
- UpdWit(W_i, a_u, PK) : given the witness W_i, the auxiliary information a_u and the public key PK, compute $w_i' = w_i a_u \bmod n^2$ then output the new witness $W_i' = (w_i', t_i)$ for the element c_i.

In the following section we show that Wang et al.'s construction is not secure.

Attack on [WWP08]

 **User**		**Manager**
		$X_0 := \emptyset$
	Insert $\mathbf{x_1}$ →	
	Delete $\mathbf{x_1}$ →	$X_1 := \{x_1\}$
	Please send $\mathbf{Upd}_{x_1,x_2}$ →	$X_2 := \emptyset$
	← $\mathbf{Upd}_{x_1,x_2}$	
With $\mathbf{Upd}_{x_1,x_2}$ **I can** update my witness $\mathbf{w}_{x_1}$		

But x_1 does not belong to X_2!

Batch Update is Impossible

- **Theorem:**

Let **Acc** be a secure accumulator scheme with deterministic **UpdWit** and **Verify** algorithms.

For an update involving **m** delete operations in a set of **N** elements, the size of the update information $\mathbf{Upd}_{X,X'}$ required by the algorithm **UpdWit** is Ω**(m log(N/m))**.

In particular if **m=N/2** we have $|\mathbf{Upd}_{X,X'}| = \Omega(m) = \Omega(N)$

Proof 1/3

 User		Manager
$X=\{x_1,x_2,\ldots,x_N\}$		$X=\{x_1,x_2,\ldots,x_N\}$
	Acc_X , $\{w_1,w_2,\ldots,w_N\}$ ←	**Compute** $Acc_X, \{w_1,w_2,\ldots,w_N\}$
		Delete $X_d:=\{x_{i_1},x_{i_2},\ldots,x_{i_m}\}$ $X' := X\setminus X_d$
	$Acc_{X'}$, $Upd_{X,X'}$ ←	**Compute** $Acc_{X'}, Upd_{X,X'}$

Proof 2/3

User

$X=\{x_1,x_2,\dots,x_N\}$
$\{w_1,\dots,w_N\}$
Acc_X, $Acc_{X'}$, $Upd_{X,X'}$

CASE 1	CASE 2
If **x** is not in **X'** => **Scheme insecure** **x** still in **X'**	If **x** is in **X'** => **Scheme incorrect** **x** not in **X'** anymore

For each element **x∈X** → **x** → **w' := UpdWit(w,Acc_X,$Acc_{X'}$,$Upd_{X,X'}$)** → **w'** valid?

YES → **CASE 1**

NO → **CASE 2**

User can reconstruct the set $\mathbf{X_d}$

Proof 3/3

- There are $\binom{\mathbf{N}}{\mathbf{m}}$ subsets of ***m*** elements in a set of ***N*** elements

- We need $\mathbf{\log\binom{N}{m} \geq m \log(N/m)}$ bits to encode $\mathbf{X_d}$

(See updated version at eprint soon for a detailed proof)

Conclusion

- Batch Update is **impossible**.
- Batch Update for accumulators with ***few*** delete operations?
- Improve the lower bound in a factor of ***k***.

Thank you!

Correction

- With negligible probability
  Bob could obtain a fake witness
  (and the scheme would still be secure)

=> The number of “good”subsets X_d is less than $\binom{N}{m}$

A more careful analysis

- Pr[X_d leads to a fake witness] $\leq$ ε(k)

  => #"Good X_d sets" $\geq \binom{N}{m}$ (1- ε(k))

  => $|Upd_{X,X'}| \geq m \log(M/m) + \log(1- \varepsilon(k))$

  => $|Upd_{X,X'}| \geq m \log(M/m) -1$

  => $|Upd_{X,X'}| = \Omega(m \log(M/m))$