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Through the alterations in the income streams provided by loans or sales, the marginal degrees of impatience for all individuals in the market are brought into equality with each other and with the market rate of interest.

Irving Fisher, *The Theory of Interest*, Macmillan, New York, 1930, 122

Introduction: Capital Markets, Consumption, and Investment

A. INTRODUCTION

The objective of this chapter is to study consumption and investment decisions made by individuals and firms. Logical development is facilitated if we begin with the simplest of all worlds, a one-person/one-good economy. The decision maker, Robinson Crusoe, must choose between consumption now and consumption in the future. Of course, the decision not to consume now is the same as investment. Thus Robinson Crusoe's decision is simultaneously one of consumption and investment. In order to decide, he needs two types of information. First, he needs to understand his own subjective trade-offs between consumption now and consumption in the future. This information is embodied in the utility and indifference curves depicted in Figs. 1.1 through 1.3. Second, he must know the feasible trade-offs between present and future consumption that are technologically possible. These are given in the investment and production opportunity sets of Figs. 1.4 and 1.5.

From the analysis of a Robinson Crusoe economy we will find that the optimal consumption/investment decision establishes a subjective interest rate for Robinson Crusoe. Shown in Fig. 1.5, it represents his (unique) optimal rate of exchange between consumption now and in the future. Thus interest rates are an integral part of consumption/investment decisions. One can think of the interest rate as the price of deferred consumption or the rate of return on investment. After the Robinson Crusoe economy we will introduce opportunities to exchange consumption across time by borrowing or lending in a multiperson economy (shown in Fig. 1.7). The introduction of these exchange opportunities results in a single market interest rate that everyone can use as a signal for making optimal consumption/investment decisions (Fig. 1.8). Furthermore, no one is worse off in an exchange economy when compared with a Robinson Crusoe economy and almost everyone is better off (Fig. 1.9). Thus an exchange economy that uses market prices (interest rates) to allocate resources across time will be seen to be superior to an economy without the price mechanism.

The obvious extension to the introductory material in this chapter is the investment decision made by firms in a multiperiod context. Managers need optimal decision rules to help in selecting those projects that maximize the wealth of shareholders. We shall see that market-determined interest rates play an important role in the corporate investment and production decisions. This material will be discussed in depth in Chapters 2 and 3.

B. CONSUMPTION AND INVESTMENT WITHOUT CAPITAL MARKETS

The answer to the question "Do capital markets benefit society?" requires that we compare a world without capital markets to one with them and show that no one is worse off and that at least one individual is better off in a world with capital markets. To make things as simple as possible, we assume that all outcomes from investment are known with certainty, that there are no transactions costs or taxes, and that decisions are made in a one-period context. Individuals are endowed with income (manna from heaven) at the beginning of the period, y_0, and at the end of the period, y_1. They must decide how much to actually consume now, C_0, and how much to invest in productive opportunities in order to provide end-of-period consumption, C_1. Every individual is assumed to prefer more consumption to less. In other words, the marginal utility of consumption is always positive. Also, we assume that the marginal utility of consumption is decreasing. The total utility curve (Fig. 1.1) shows the utility of consumption at the beginning of the period, assuming that the second-period consumption is held constant. Changes in consumption have been marked off in equal increments along the horizontal axis. Note that equal increases in consumption cause total utility to increase (marginal utility is positive), but that the increments in utility become smaller and smaller (marginal utility is decreasing). We can easily construct a similar graph to represent the utility of end-of-period consumption, $U(C_1)$. When combined with Fig. 1.1, the result (the three-dimensional graph shown in Fig. 1.2) provides a description of trade-offs between consumption at the beginning of the period, C_0, and consumption at the end of the period, C_1. The dashed lines represent contours along the utility surface where various combinations of C_0 and C_1 provide the same total utility (measured along the vertical axis). Since all points along the same contour (e.g., points A and B) have equal total utility, the individual will be indifferent with respect to them. Therefore the contours are called *indifference curves*.

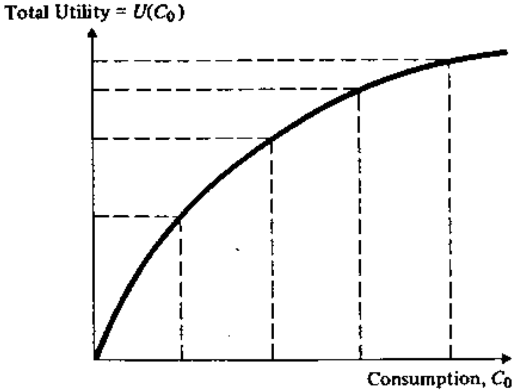

Figure 1.1
Total utility of consumption.

Looking at Fig. 1.2 from above, we can project the indifference curves onto the consumption argument plane (i.e., the plane formed by the C_0, C_1 axes in Fig. 1.3). To reiterate, all combinations of consumption today and consumption tomorrow that lie on the same indifference curve have the same total utility. The decision maker whose indifference curves are depicted in Fig. 1.3 would be indifferent as to point A with consumption (C_{0a}, C_{1a}) and point B with consumption (C_{0b}, C_{1b}). Point A has more consumption at the end of the period but less consumption at the beginning than point B does. Point D has more consumption in both periods than do either points A or B. Point D lies on an indifference curve with higher utility than points A and B; hence curves to the northeast have greater total utility.

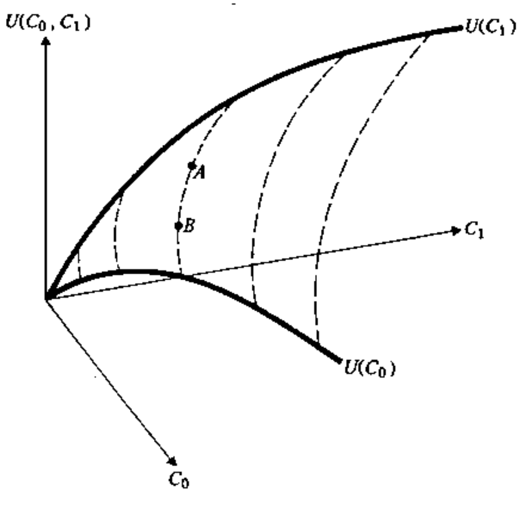

Figure 1.2
Trade-offs between beginning and end-of-period consumption.

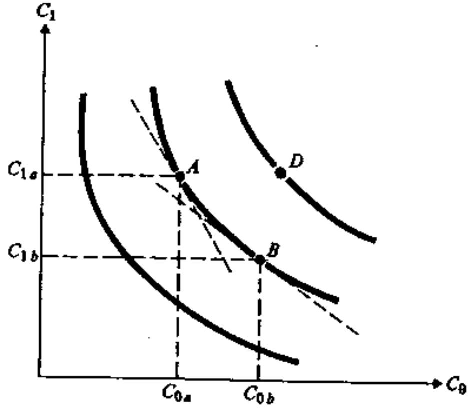

Figure 1.3
Indifference curves representing the time preference of consumption.

The slope of the straight line just tangent to the indifference curve at point B measures the rate of trade-off between C_0 and C_1 at point B. This trade-off is called the *marginal rate of substitution* (MRS) between consumption today and consumption tomorrow. It also reveals the decision maker's subjective rate of time preference, r_i, at point B. We can think of the subjective rate of time preference as an interest rate because it measures the rate of substitution between consumption bundles over time. It reveals how many extra units of consumption tomorrow must be received in order to give up one unit of consumption today and still have the same total utility. Mathematically, it is expressed as[1]

$$\text{MRS}_{C_1}^{C_0} = \left.\frac{\partial C_1}{\partial C_0}\right|_{U=\text{const.}} = -(1 + r_i). \qquad (1.1)$$

Note that the subjective rate of time preference is greater at point A than at point B. The individual has less consumption today at point A and will therefore demand relatively more future consumption in order to have the same total utility.

Thus far we have described preference functions that tell us how individuals will make choices among consumption bundles over time. What happens if we introduce productive opportunities that allow a unit of current savings/investment to be turned into more than one unit of future consumption? We assume that each individual in the economy has a schedule of productive investment opportunities that can be arranged from the highest rate of return down to the lowest (Fig. 1.4). Although we have chosen to graph the investment opportunities schedule as a straight line, any decreasing function would do. This implies diminishing marginal returns to investment because the more an individual invests, the lower the rate of return on the marginal investment. Also, all investments are assumed independent of one another and perfectly divisible.

[1] Equation (1.1) can be read as follows: The marginal rate of substitution between consumption today and end-of-period consumption, $\text{MRS}_{C_1}^{C_0}$, is equal to the slope of a line tangent to an indifference curve given constant total utility $[\partial C_1/\partial C_0]|_{U=\text{const.}}$. This in turn is equal to the individual's subjective rate of time preference, $-(1 + r_i)$.

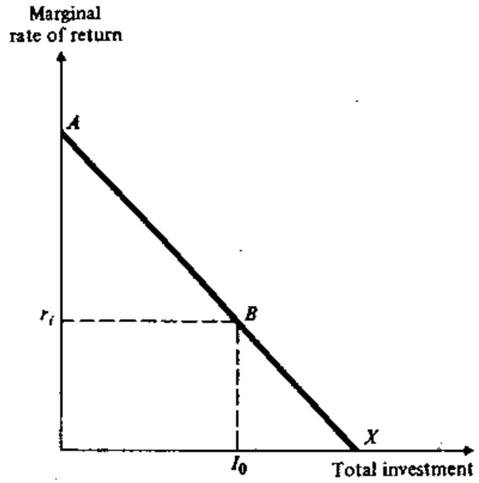

Figure 1.4
An individual's schedule of productive investment opportunities.

An individual will make all investments in productive opportunities that have rates of return higher than his or her subjective rate of time preference, r_i. This can be demonstrated if we transform the schedule of productive investment opportunities into the consumption argument plane (Fig. 1.5).[2] The slope of a line tangent to curve ABX in Fig. 1.5 is the rate at which a dollar of consumption foregone today is transformed by productive investment into a dollar of consumption tomorrow. It is the

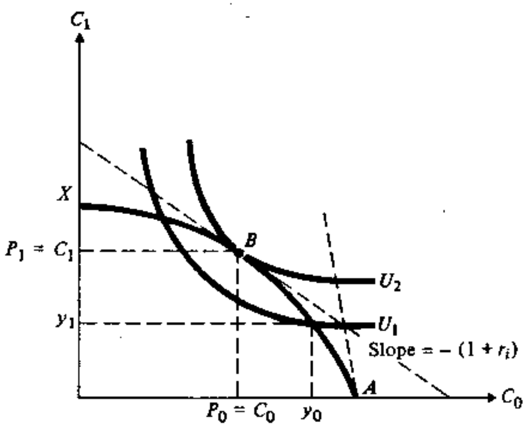

Figure 1.5
The production opportunity set.

[2] See Problem 1.6 at the end of the chapter for an example of how to make the transition between the schedule of productive investment opportunities and the consumption argument plane.

marginal rate of transformation (MRT) offered by the production/investment opportunity set. The line tangent to point A has the highest slope in Fig. 1.5 and represents the highest rate of return at point A in Fig. 1.4. An individual endowed with a resource bundle (y_0, y_1) that has utility U_1 can move along the production opportunity set to point B, where the indifference curve is tangent to it and he or she receives the maximum attainable utility, U_2. Because current consumption, C_0, is less than the beginning-of-period endowment, y_0, the individual has chosen to invest. The amount of investment is $y_0 - C_0$. Of course, if $C_0 > y_0$, he or she will disinvest.

Note that the marginal rate of return on the last investment made (i.e., MRT, the slope of a line tangent to the investment opportunity set at point B) is exactly equal to the investor's subjective time preference (i.e., MRS, the slope of a line tangent to his or her indifference curve, also at point B). In other words, the investor's subjective marginal rate of substitution is equal to the marginal rate of transformation offered by the production opportunity set:

$$\text{MRS} = \text{MRT}.$$

This will always be true in a Robinson Crusoe world where there are no capital markets, i.e., no opportunities to exchange. The individual decision maker starts with an initial endowment (y_0, y_1) and compares the marginal rate of return on a dollar of productive investment (or disinvestment) with his or her subjective time preference. If the rate on investment is greater (as it is in Fig. 1.5), he or she will gain utility by making the investment. This process continues until the rate of return on the last dollar of productive investment just equals the rate of subjective time preference (at point B). Note that at point B the individual's consumption in each time period is exactly equal to the output from production, i.e., $P_0 = C_0$ and $P_1 = C_1$.

Without the existence of capital markets, individuals with the same endowment and the same investment opportunity set may choose completely different investments because they have different indifference curves. This is shown in Fig. 1.6. Individual

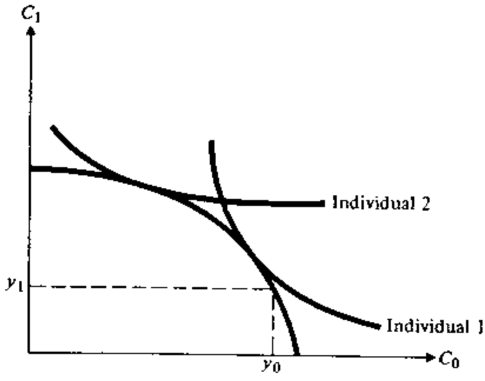

Figure 1.6
Individuals with different indifference curves choose different production/consumption patterns.

2, who has a lower rate of time preference (Why?), will choose to invest more than individual 1.

C. CONSUMPTION AND INVESTMENT WITH CAPITAL MARKETS

A Robinson Crusoe economy is characterized by the fact that there are no opportunities to exchange intertemporal consumption among individuals. What happens if—instead of one person—many individuals are said to exist in the economy? Intertemporal exchange of consumption bundles will be represented by the opportunity to borrow or lend unlimited amounts at r, a market-determined rate of interest.[3]

Financial markets facilitate the transfer of funds between lenders and borrowers. Assuming that interest rates are positive, any amount of funds lent today will return interest plus principal at the end of the period. Ignoring production for the time being, we can graph borrowing and lending opportunities along the *capital market line* in Fig. 1.7 (line W_0ABW_1). With an initial endowment of (y_0, y_1) that has utility equal to U_1, we can reach any point along the market line by borrowing or lending at the market interest rate plus repaying the principal amount, X_0. If we designate the future value as X_1, we can write that the future value is equal to the principal amount plus interest earned,

$$X_1 = X_0 + rX_0, \qquad X_1 = (1 + r)X_0.$$

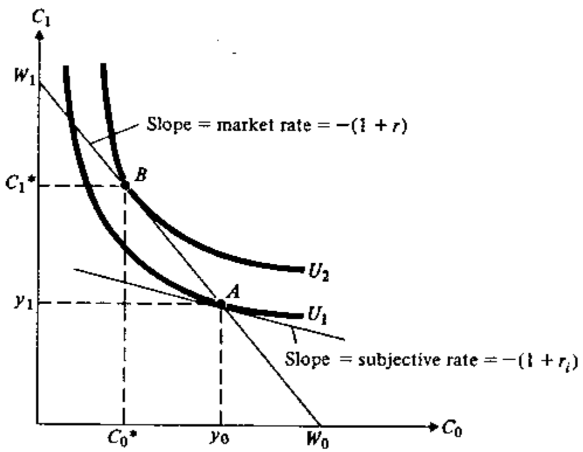

Figure 1.7
The capital market line.

[3] The market rate of interest is provided by the solution to a general equilibrium problem. For simplicity, we assume that the market rate of interest is a given.

Similarly, the present value, W_0, of our initial endowment, (y_0, y_1), is the sum of current income, y_0, and the present value of our end-of-period income, $y_1(1 + r)^{-1}$:

$$W_0 = y_0 + \frac{y_1}{(1 + r)}. \tag{1.2}$$

Referring to Fig. 1.7, we see that with endowment (y_0, y_1) we will maximize utility by moving along the market line to the point where our subjective time preference equals the market interest rate. Point B represents the consumption bundle (C_0^*, C_1^*) on the highest attainable indifference curve. At the initial endowment (point A), our subjective time preference, represented by the slope of a line tangent to the indifference curve at point A, is less than the market rate of return. Therefore we will desire to lend because the capital market offers a rate of return higher than what we subjectively require. Ultimately, we reach a consumption decision (C_0^*, C_1^*) where we maximize utility. The utility, U_2, at point B is greater than the utility, U_1, at our initial endowment, point A. The present value of this consumption bundle is also equal to our wealth, W_0:

$$W_0 = C_0^* + \frac{C_1^*}{1 + r}. \tag{1.3}$$

This can be rearranged to give the equation for the capital market line:

$$C_1^* = W_0(1 + r) - (1 + r)C_0^*, \tag{1.4}$$

and since $W_0(1 + r) = W_1$, we have

$$C_1^* = W_1 - (1 + r)C_0^*. \tag{1.5}$$

Thus the capital market line in Fig. 1.7 has an intercept at W_1 and a slope of $-(1 + r)$. Also note that by equating (1.2) and (1.3) we see that the present value of our endowment equals the present value of our consumption, and both are equal to our wealth, W_0. Moving along the capital market line does not change one's wealth, but it does offer a pattern of consumption that has higher utility.

What happens if the production/consumption decision takes place in a world where capital markets facilitate the exchange of funds at the market rate of interest? Figure 1.8 combines production possibilities with market exchange possibilities. With the family of indifference curves U_1, U_2, and U_3 and endowment (y_0, y_1) at point A, what actions will we take in order to maximize our utility? Starting at point A, we can move either along the production opportunity set or along the capital market line. Both alternatives offer a higher rate of return than our subjective time preference, but production offers the higher return, i.e., a steeper slope. Therefore we choose to invest and move along the production opportunity frontier. Without the opportunity to borrow or lend along the capital market line, we would stop investing at point D, where the marginal return on productive investment equals our subjective time preference. This was the result shown for consumption and investment in a Robinson Crusoe world without capital markets in Fig. 1.5. At this point, our level of utility

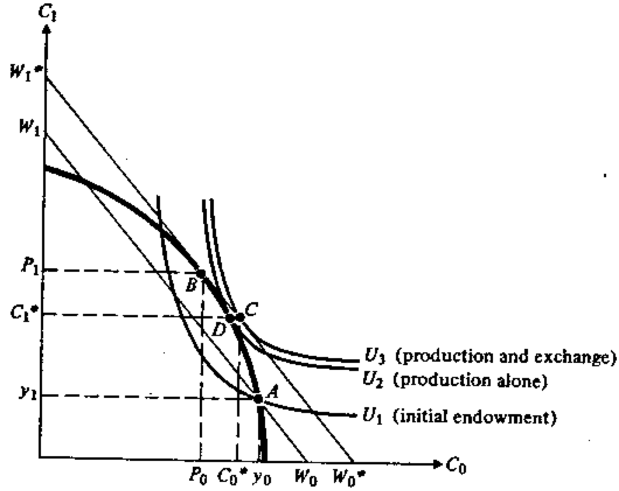

Figure 1.8
Production and consumption with capital markets.

has increased from U_1 to U_2. However, with the opportunity to borrow, we can actually do better. Note that at point D the borrowing rate, represented by the slope of the capital market line, is less than the rate of return on the marginal investment, which is the slope of the production opportunity set at point D. Since further investment returns more than the cost of borrowed funds, we will continue to invest until the marginal return on investment is equal to the borrowing rate at point B. At point B, we receive the output from production (P_0, P_1), and the present value of our wealth is W_0^* instead of W_0. Furthermore, we can now reach any point on the market line. Since our time preference at point B is greater than the market rate of return, we will consume more than P_0, which is the current payoff from production. By borrowing, we can reach point C on the capital market line. Our optimal consumption is found, as before, where our subjective time preference just equals the market rate of return. Our utility has increased from U_1 at point A (our initial endowment) to U_2 at point D (the Robinson Crusoe solution) to U_3 at point C (the exchange economy solution). We are clearly better off when capital markets exist since $U_3 > U_2$.

The decision process that takes place with production opportunities and capital market exchange opportunities occurs in two separate and distinct steps: (1) first, choose the optimal production decision by taking on projects until the marginal rate of return on investment equals the objective market rate; (2) then choose the optimal consumption pattern by borrowing or lending along the capital market line to equate your subjective time preference with the market rate of return. The separation of the investment (step 1) and consumption (step 2) decisions is known as the Fisher separation theorem.

Fisher separation theorem. Given perfect and complete capital markets, the production decision is governed solely by an objective market criterion (represented by maximizing attained wealth) without regard to individuals' subjective preferences that enter into their consumption decisions.

An important implication for corporate policy is that the investment decision can be delegated to managers. Given the same opportunity set, every investor will make the same production decision (P_0, P_1) regardless of the shape of his or her indifference curves. This is shown in Fig. 1.9. Both investor 1 and investor 2 will direct the manager of their firm to choose production combination (P_0, P_1). They can then take the output of the firm and adapt it to their own subjective time preferences by borrowing or lending in the capital market. Investor 1 will choose to consume more than his or her share of current production (point *A*) by borrowing today in the capital market and repaying out of his or her share of future production. Alternately, investor 2 will lend because he or she consumes less than his or her share of current production. Either way, they are both better off with a capital market. The optimal production decision is separated from individual utility preferences. Without capital market opportunities to borrow or lend, investor 1 would choose to produce at point *Y*, which has lower utility. Similarly, investor 2 would be worse off at point *X*.

In equilibrium, the marginal rate of substitution for all investors is equal to the market rate of interest, and this in turn is equal to the marginal rate of transformation for productive investment. Mathematically, the marginal rates of substitution for investors *i* and *j* are

$$\text{MRS}_i = \text{MRS}_j = -(1 + r) = \text{MRT}.$$

Thus all individuals use the same time value of money (i.e., the same market-determined objective interest rate) in making their production/investment decisions.

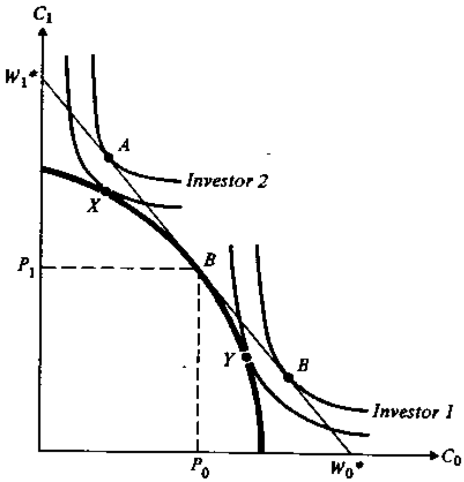

Figure 1.9
The investment decision is independent of individual preferences.

The importance of capital markets cannot be overstated. They allow the efficient transfer of funds between borrowers and lenders. Individuals who have insufficient wealth to take advantage of all their investment opportunities that yield rates of return higher than the market rate are able to borrow funds and invest more than they would without capital markets. In this way, funds can be efficiently allocated from individuals with few productive opportunities and great wealth to individuals with many opportunities and insufficient wealth. As a result, all (borrowers and lenders) are better off than they would have been without capital markets.

D. MARKETPLACES AND TRANSACTIONS COSTS

The foregoing discussion has demonstrated the advantages of capital markets for funds allocation in a world without transactions costs. In such a world, there is no need for a central location for exchange; that is, there is no need for a marketplace per se. But let us assume that we have a primitive economy with N producers, each making a specialized product and consuming a bundle of all N consumption goods. Given no marketplace, bilateral exchange is necessary. During a given time period, each visits the other in order to exchange goods. The cost of each leg of a trip is T dollars. If there are five individuals and five consumption goods in this economy, then individual 1 makes four trips, one to each of the other four producers. Individual 2 makes three trips, and so on. Altogether, there are $[N(N-1)]/2 = 10$ trips, at a total cost of $10T$ dollars. This is shown in Fig. 1.10. If an entrepreneur establishes a central marketplace and carries an inventory of each of the N products, as shown in Fig. 1.11, the total number of trips can be reduced to five, with a total cost of $5T$ dollars. Therefore if the entrepreneur has a total cost (including the cost of living) of less than $10T - 5T$ dollars, he or she can profitably establish a marketplace and everyone will be better off.[4]

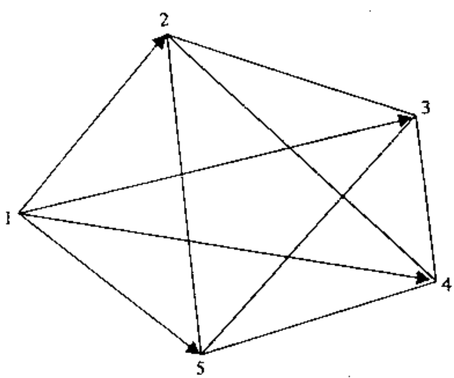

Figure 1.10
A primitive exchange economy with no central marketplace.

[4] In general, for N individuals making two-way exchanges, there are $\binom{N}{2} = N(N-1)/2$ trips. With a marketplace the number of trips is reduced to N. Therefore the savings is $[N(N-1)/2 - N]T$.

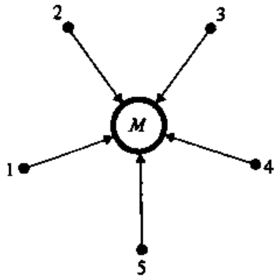

Figure 1.11
The productivity of a central marketplace.

This example provides a simple explanation for the productivity of marketplaces. Among other things, they serve to efficiently reduce transactions costs. Later on, we shall refer to this fact as the *operational efficiency* of capital markets. The lower the transactions costs are, the more operationally efficient a market can be.

E. TRANSACTIONS COSTS AND THE BREAKDOWN OF SEPARATION

If transactions costs are nontrivial, financial intermediaries and marketplaces will provide a useful service. In such a world, the borrowing rate will be greater than the lending rate. Financial institutions will pay the lending rate for money deposited with them and then issue funds at a higher rate to borrowers. The difference between the borrowing and lending rates represents their (competitively determined) fee for the economic service provided. Different borrowing and lending rates will have the effect

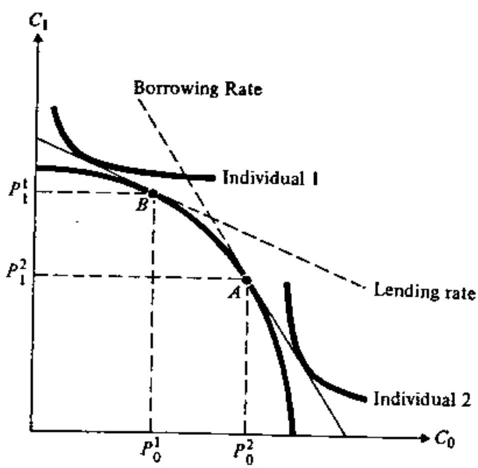

Figure 1.12
Markets with different borrowing and lending rates.

of invalidating the Fisher separation principle. As shown in Fig. 1.12, individuals with different indifference curves will now choose different levels of investment. Without a single market rate they will not be able to delegate the investment decision to the manager of their firm. Individual 1 would direct the manager to use the lending rate and invest at point *B*. Individual 2 would use the borrowing rate and choose point *A*. A third individual might choose investments between points *A* and *B*, where his or her indifference curve is directly tangent to the production opportunity set.

The theory of finance is greatly simplified if we assume that capital markets are perfect. Obviously they are not. The relevant question then is whether the theories that assume frictionless markets fit reality well enough to be useful or whether they need to be refined in order to provide greater insights. This is an empirical question that will be addressed later on in the text.

Throughout most of this text we shall adopt the convenient and simplifying assumption that capital markets are perfect. The only major imperfections to be considered in detail are the impact of corporate and personal taxes and information asymmetries. The effects of taxes and imperfect information are certainly nontrivial, and as we shall see, they do change the predictions of many models of financial policy.

SUMMARY

The rest of the text follows almost exactly the same logic as this chapter, except that from Chapter 4 onward it focuses on decision making under uncertainty. The first step is to develop indifference curves to model individual decision making in a world with uncertainty. Chapter 4 is analogous to Fig. 1.3. It will describe a theory of choice under uncertainty. Next, the portfolio opportunity set, which represents choices among combinations of risky assets, is developed. Chapters 5 and 6 are similar to Fig. 1.5. They describe the objects of choice—the portfolio opportunity set. The tangency between the indifference curves of a risk-averse investor and his or her opportunity set provides a theory of individual choice in a world without capital markets (this is discussed in Chapter 6). Finally, in Chapter 7, we introduce the opportunity to borrow and lend at a riskless rate and develop models of capital market equilibrium. Chapter 7 follows logic similar to Fig. 1.8. In fact, we show that a type of separation principle (two-fund separation) obtains, given uncertainty and perfect capital markets. Chapters 10 and 11 take a careful look at the meaning of efficient capital markets and at empirical evidence that relates to the question of how well the perfect capital market assumption fits reality. The remainder of the book, following Chapter 11, applies financial theory to corporate policy decisions.

PROBLEM SET

1.1 Graphically demonstrate the Fisher separation theorem for the case where an individual ends up lending in financial markets. Label the following points on the graph: initial wealth, W_0; optimal production/investment (P_0, P_1); optimal consumption (C_0^*, C_1^*); present value of final wealth, W_0^*.

1.2 Graphically analyze the effect of an exogenous decrease in the interest rate on (a) the utility of borrowers and lenders, (b) the present wealth of borrowers and lenders, and (c) the investment in real assets.

1.3 The interest rate cannot fall below the net rate from storage. True or false? Why?

1.4 Graphically illustrate the decision-making process faced by an individual in a Robinson Crusoe economy where (a) storage is the only investment opportunity and (b) there are no capital markets.

1.5 Suppose that the investment opportunity set has N projects, all of which have the same rate of return, R^*. Graph the investment set.

1.6 Suppose your production opportunity set in a world with perfect certainty consists of the following possibilities:

Project	*Investment Outlay*	*Rate of Return*
A	$1,000,000	8%
B	1,000,000	20
C	2,000,000	4
D	3,000,000	30

a) Graph the production opportunity set in a C_0, C_1 framework.

b) If the market rate of return is 10%, draw in the capital market line for the optimal investment decision.

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